Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/7.1.2-d-x-^m-a+b-arcsinh-c-x-^n

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May 24, 2020 Compiled on May 24, 2020 at 4:00pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [156]. This is test number [186].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (156)	% 0. (0)
Mathematica	% 100. (156)	% 0. (0)
Maple	% 69.87 (109)	% 30.13 (47)
Maxima	% 31.41 (49)	% 68.59 (107)
Fricas	% 27.56 (43)	% 72.44 (113)
Sympy	% 28.85 (45)	% 71.15 (111)
Giac	% 28.85 (45)	% 71.15 (111)

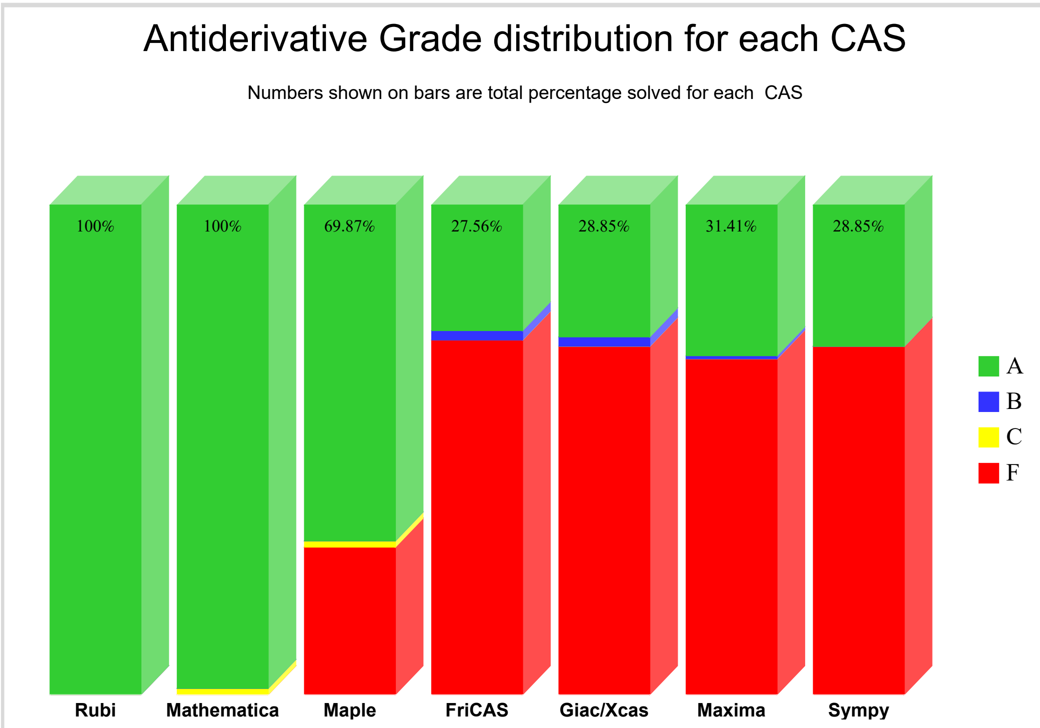
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

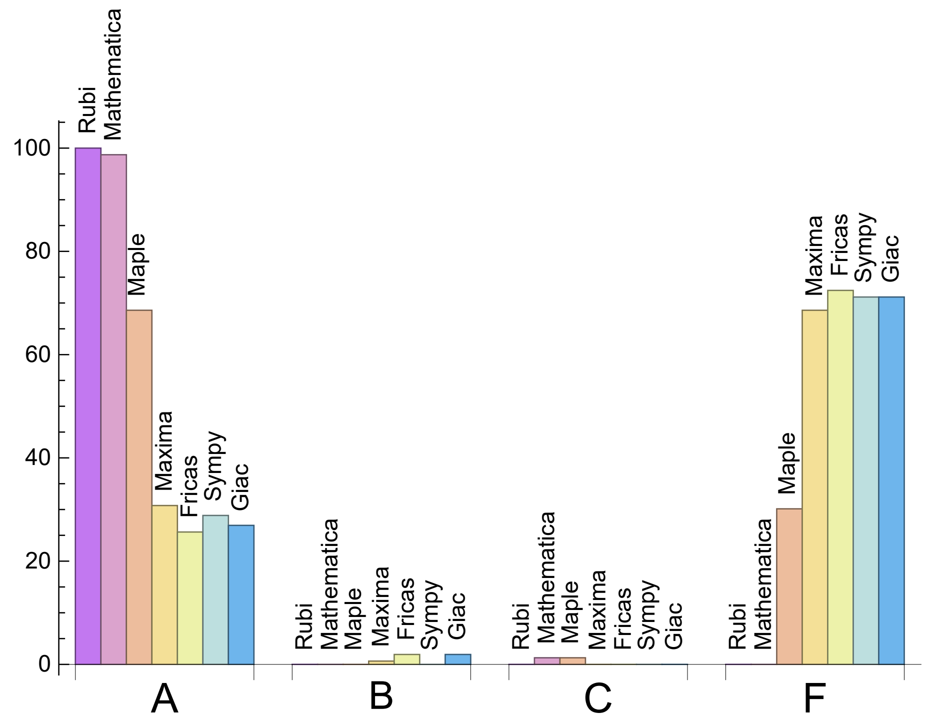
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	98.72	0.	1.28	0.
Maple	68.59	0.	1.28	30.13
Maxima	30.77	0.64	0.	68.59
Fricas	25.64	1.92	0.	72.44
Sympy	28.85	0.	0.	71.15
Giac	26.92	1.92	0.	71.15

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	101.14	0.82	84.5	1.
Mathematica	0.42	91.44	0.75	75.	0.85
Maple	0.07	74.68	0.83	56.	0.91
Maxima	0.55	48.73	0.61	0.	0.
Fricas	1.3	152.6	1.84	134.	2.
Sympy	1.71	50.13	0.44	0.	0.
Giac	0.68	63.33	0.88	0.	0.

1.4 list of integrals that has no closed form antiderivative

{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {18, 20, 28, 29, 30, 31, 39, 41, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

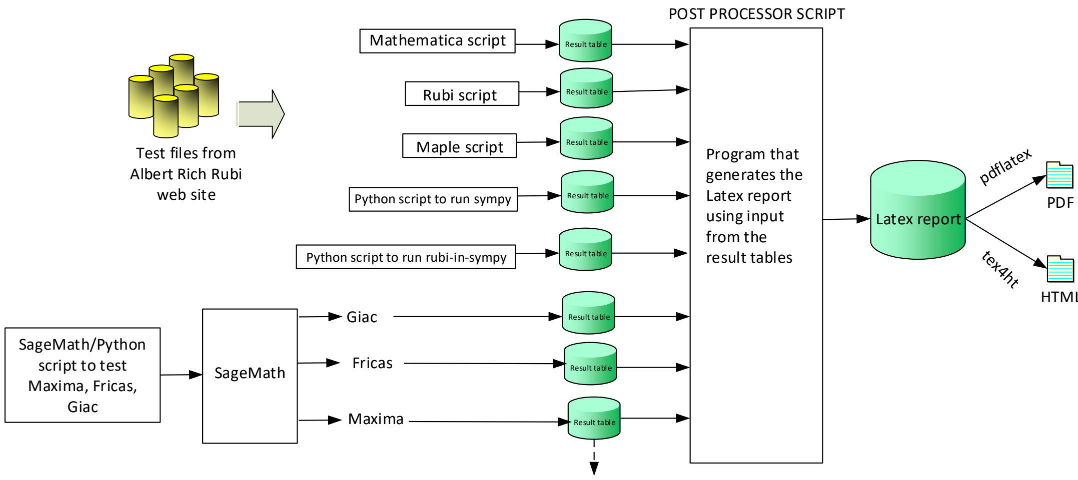
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { 11, 40 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 83, 84, 85, 89, 90, 91, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135 }

B grade: { }

C grade: { 132, 133 }

F grade: { 74, 75, 76, 80, 81, 82, 86, 87, 88, 92, 93, 94, 99, 100, 101, 105, 106, 107, 111, 112, 113, 119, 120, 129, 130, 131, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135

B grade: { 15 }

C grade: { }

F grade: { 6, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 117, 118, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 10, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 117, 118, 121, 122, 128, 134, 135 }

B grade: { 7, 9, 11 }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 97, 98, 104, 110, 117, 118, 121, 122, 125, 126, 127, 128, 134, 135 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 123, 124, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 14, 16, 22, 24, 26, 33, 35, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 126, 127, 134, 135 }

B grade: { 7, 19, 21 }

C grade: { }

F grade: { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 123, 124, 125, 128, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	69	92	135	70	90
normalized size	1	1.	0.69	0.96	1.28	1.88	0.97	1.25
time (sec)	N/A	0.044	0.035	0.031	1.137	1.838	2.166	1.302

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	49	58	96	131	61	108
normalized size	1	1.	0.73	0.87	1.43	1.96	0.91	1.61
time (sec)	N/A	0.026	0.017	0.005	1.179	1.863	1.16	1.377

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	50	65	113	48	70
normalized size	1	1.	0.79	0.96	1.25	2.17	0.92	1.35
time (sec)	N/A	0.035	0.024	0.005	1.164	1.78	0.551	1.317

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	40	39	69	109	37	92
normalized size	1	1.	0.91	0.89	1.57	2.48	0.84	2.09
time (sec)	N/A	0.016	0.01	0.003	1.193	1.768	0.237	1.287

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	34	78	20	47
normalized size	1	1.	1.	1.04	1.36	3.12	0.8	1.88
time (sec)	N/A	0.008	0.007	0.002	1.108	1.753	0.153	1.352

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	94	0	0	0	0
normalized size	1	1.	1.	2.19	0.	0.	0.	0.
time (sec)	N/A	0.06	0.003	0.148	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	32	212	0	76
normalized size	1	1.	1.	1.11	1.19	7.85	0.	2.81
time (sec)	N/A	0.021	0.002	0.005	1.123	1.946	0.	1.39

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	28	37	36	88	0	68
normalized size	1	1.	0.85	1.12	1.09	2.67	0.	2.06
time (sec)	N/A	0.014	0.006	0.004	1.184	1.948	0.	1.427

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	61	273	0	104
normalized size	1	1.	1.	0.94	1.13	5.06	0.	1.93
time (sec)	N/A	0.032	0.01	0.006	1.119	2.03	0.	1.378

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	40	56	66	109	0	104
normalized size	1	1.	0.71	1.	1.18	1.95	0.	1.86
time (sec)	N/A	0.02	0.012	0.003	1.175	1.83	0.	1.353

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	70	88	302	0	128
normalized size	1	1.	0.64	0.91	1.14	3.92	0.	1.66
time (sec)	N/A	0.045	0.011	0.006	1.164	1.958	0.	1.428

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	153	134	234	114	153
normalized size	1	1.	0.64	1.31	1.15	2.	0.97	1.31
time (sec)	N/A	0.188	0.064	0.131	1.159	1.786	4.139	1.402

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	72	118	173	205	90	0
normalized size	1	1.	0.75	1.23	1.8	2.14	0.94	0.
time (sec)	N/A	0.165	0.044	0.035	1.269	1.82	2.28	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	59	92	95	188	76	120
normalized size	1	1.	0.74	1.15	1.19	2.35	0.95	1.5
time (sec)	N/A	0.123	0.058	0.027	1.235	1.804	1.127	1.435

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	59	135	166	51	0
normalized size	1	1.	0.9	1.	2.29	2.81	0.86	0.
time (sec)	N/A	0.092	0.028	0.026	1.127	2.006	0.541	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	43	134	32	84
normalized size	1	1.	1.	1.06	1.26	3.94	0.94	2.47
time (sec)	N/A	0.045	0.014	0.023	1.182	2.101	0.218	1.455

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	151	0	0	0	0
normalized size	1	1.	1.	2.52	0.	0.	0.	0.
time (sec)	N/A	0.097	0.006	0.036	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	75	107	0	0	0	0
normalized size	1	1.	1.5	2.14	0.	0.	0.	0.
time (sec)	N/A	0.1	0.192	0.049	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	67	53	157	0	149
normalized size	1	1.	1.	1.56	1.23	3.65	0.	3.47
time (sec)	N/A	0.08	0.031	0.076	1.192	2.174	0.	1.482

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	125	144	0	0	0	0
normalized size	1	1.	1.26	1.45	0.	0.	0.	0.
time (sec)	N/A	0.163	0.49	0.092	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	99	96	194	0	200
normalized size	1	1.	0.75	1.16	1.13	2.28	0.	2.35
time (sec)	N/A	0.138	0.059	0.106	1.225	2.162	0.	1.594

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	120	222	223	359	196	243
normalized size	1	1.	0.62	1.14	1.14	1.84	1.01	1.25
time (sec)	N/A	0.366	0.069	0.04	1.084	2.163	7.481	1.599

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	110	168	0	327	160	0
normalized size	1	1.	0.67	1.03	0.	2.01	0.98	0.
time (sec)	N/A	0.298	0.069	0.034	0.	2.142	4.269	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	93	136	157	281	128	190
normalized size	1	1.	0.7	1.03	1.19	2.13	0.97	1.44
time (sec)	N/A	0.221	0.054	0.028	1.141	2.116	2.24	1.566

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	88	0	261	92	0
normalized size	1	1.	0.82	0.91	0.	2.69	0.95	0.
time (sec)	N/A	0.153	0.044	0.025	0.	2.081	1.119	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	77	205	54	132
normalized size	1	1.	1.	0.95	1.33	3.53	0.93	2.28
time (sec)	N/A	0.081	0.016	0.027	1.308	2.02	0.518	1.43

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	204	0	0	0	0
normalized size	1	1.	1.	2.46	0.	0.	0.	0.
time (sec)	N/A	0.112	0.007	0.036	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	117	162	0	0	0	0
normalized size	1	1.	1.39	1.93	0.	0.	0.	0.
time (sec)	N/A	0.159	0.116	0.049	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	80	149	0	0	0	0
normalized size	1	1.	0.86	1.6	0.	0.	0.	0.
time (sec)	N/A	0.168	0.317	0.068	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	268	228	0	0	0	0
normalized size	1	1.	1.77	1.51	0.	0.	0.	0.
time (sec)	N/A	0.277	2.219	0.092	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	107	210	0	0	0	0
normalized size	1	1.	0.67	1.32	0.	0.	0.	0.
time (sec)	N/A	0.286	0.607	0.106	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	165	319	0	497	269	0
normalized size	1	1.	0.6	1.16	0.	1.8	0.97	0.
time (sec)	N/A	0.856	0.09	0.16	0.	2.14	21.536	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	148	272	271	466	241	293
normalized size	1	1.	0.61	1.11	1.11	1.91	0.99	1.2
time (sec)	N/A	0.657	0.081	0.04	1.236	2.144	12.465	1.817

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	133	208	0	402	190	0
normalized size	1	1.	0.69	1.07	0.	2.07	0.98	0.
time (sec)	N/A	0.504	0.07	0.036	0.	2.108	7.516	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	112	165	193	358	158	230
normalized size	1	1.	0.69	1.02	1.19	2.21	0.98	1.42
time (sec)	N/A	0.363	0.07	0.03	1.235	2.143	4.244	1.809

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	94	105	0	316	104	0
normalized size	1	1.	0.85	0.95	0.	2.87	0.95	0.
time (sec)	N/A	0.239	0.045	0.026	0.	2.051	2.141	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	65	99	262	65	169
normalized size	1	1.	1.	0.97	1.48	3.91	0.97	2.52
time (sec)	N/A	0.125	0.019	0.026	1.138	2.055	1.033	1.589

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	257	0	0	0	0
normalized size	1	1.	1.	2.65	0.	0.	0.	0.
time (sec)	N/A	0.122	0.008	0.036	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	161	217	0	0	0	0
normalized size	1	1.	1.34	1.81	0.	0.	0.	0.
time (sec)	N/A	0.188	0.226	0.05	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	113	208	0	0	0	0
normalized size	1	1.	1.05	1.93	0.	0.	0.	0.
time (sec)	N/A	0.207	0.259	0.068	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	355	372	0	0	0	0
normalized size	1	1.	1.59	1.67	0.	0.	0.	0.
time (sec)	N/A	0.393	2.538	0.091	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	0
normalized size	1	1.	0.73	0.73	0.	0.	0.	0.
time (sec)	N/A	0.1	0.012	0.041	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	0
normalized size	1	1.	0.77	0.77	0.	0.	0.	0.
time (sec)	N/A	0.083	0.11	0.033	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	0
normalized size	1	1.	0.76	0.76	0.	0.	0.	0.
time (sec)	N/A	0.084	0.01	0.025	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	0
normalized size	1	1.	0.83	0.83	0.	0.	0.	0.
time (sec)	N/A	0.068	0.072	0.024	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	0	0	0	0
normalized size	1	1.	0.81	0.81	0.	0.	0.	0.
time (sec)	N/A	0.067	0.006	0.02	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	0
normalized size	1	1.	1.	0.93	0.	0.	0.	0.
time (sec)	N/A	0.038	0.02	0.03	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0
normalized size	1	1.	1.	1.11	0.	0.	0.	0.
time (sec)	N/A	0.018	0.009	0.019	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.172	0.075	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.803	0.086	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	85	104	0	0	0	0
normalized size	1	1.	1.04	1.27	0.	0.	0.	0.
time (sec)	N/A	0.081	0.28	0.043	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	78	78	0	0	0	0
normalized size	1	1.	1.11	1.11	0.	0.	0.	0.
time (sec)	N/A	0.066	0.041	0.038	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	80	0	0	0	0
normalized size	1	1.	0.88	1.18	0.	0.	0.	0.
time (sec)	N/A	0.066	0.202	0.028	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	54	0	0	0	0
normalized size	1	1.	1.	0.96	0.	0.	0.	0.
time (sec)	N/A	0.049	0.026	0.025	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	56	0	0	0	0
normalized size	1	1.	0.91	1.04	0.	0.	0.	0.
time (sec)	N/A	0.047	0.166	0.024	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	28	0	0	0	0
normalized size	1	1.	0.86	0.76	0.	0.	0.	0.
time (sec)	N/A	0.025	0.003	0.026	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	30	0	0	0	0
normalized size	1	1.	0.91	0.88	0.	0.	0.	0.
time (sec)	N/A	0.083	0.048	0.023	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.729	0.06	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	4.768	0.082	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	102	120	0	0	0	0
normalized size	1	1.	1.05	1.24	0.	0.	0.	0.
time (sec)	N/A	0.354	0.14	0.039	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	82	0	0	0	0
normalized size	1	1.	0.84	1.	0.	0.	0.	0.
time (sec)	N/A	0.304	0.203	0.03	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	81	0	0	0	0
normalized size	1	1.	0.79	1.	0.	0.	0.	0.
time (sec)	N/A	0.253	0.137	0.029	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	43	0	0	0	0
normalized size	1	1.	0.98	0.68	0.	0.	0.	0.
time (sec)	N/A	0.171	0.049	0.03	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	42	0	0	0	0
normalized size	1	1.	0.94	0.84	0.	0.	0.	0.
time (sec)	N/A	0.083	0.018	0.022	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.54	0.06	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	4.992	0.084	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	169	0	0	0	0
normalized size	1	1.	1.01	1.09	0.	0.	0.	0.
time (sec)	N/A	0.32	0.328	0.046	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	105	114	0	0	0	0
normalized size	1	1.	0.74	0.81	0.	0.	0.	0.
time (sec)	N/A	0.283	0.373	0.031	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	99	115	0	0	0	0
normalized size	1	1.	0.72	0.83	0.	0.	0.	0.
time (sec)	N/A	0.306	0.29	0.031	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	84	60	0	0	0	0
normalized size	1	1.	0.88	0.63	0.	0.	0.	0.
time (sec)	N/A	0.163	0.116	0.025	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	61	0	0	0	0
normalized size	1	1.	0.91	0.8	0.	0.	0.	0.
time (sec)	N/A	0.15	0.058	0.023	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.801	0.06	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	8.822	0.083	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	161	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	0.045	0.241	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	101	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	0.036	0.112	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	101	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	0.034	0.097	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	52	75	0	0	0	0
normalized size	1	1.	0.56	0.81	0.	0.	0.	0.
time (sec)	N/A	0.191	0.033	0.105	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	45	42	0	0	0	0
normalized size	1	1.	0.85	0.79	0.	0.	0.	0.
time (sec)	N/A	0.107	0.047	0.068	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.333	0.062	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	152	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.713	0.117	0.206	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	199	102	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.487	0.035	0.099	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	102	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.371	0.034	0.099	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	52	102	0	0	0	0
normalized size	1	1.	0.43	0.84	0.	0.	0.	0.
time (sec)	N/A	0.219	0.021	0.091	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	47	65	0	0	0	0
normalized size	1	1.	0.58	0.8	0.	0.	0.	0.
time (sec)	N/A	0.119	0.028	0.074	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.315	0.058	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	152	0	0	0	0	0
normalized size	1	1.	0.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.996	0.109	0.204	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	101	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.711	0.037	0.099	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	101	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.551	0.036	0.1	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	52	136	0	0	0	0
normalized size	1	1.	0.34	0.89	0.	0.	0.	0.
time (sec)	N/A	0.337	0.032	0.095	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	45	78	0	0	0	0
normalized size	1	1.	0.48	0.83	0.	0.	0.	0.
time (sec)	N/A	0.184	0.048	0.072	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.314	0.057	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.102	0.214	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.076	0.106	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	99	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.072	0.118	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	52	37	0	0	0	0
normalized size	1	1.	0.83	0.59	0.	0.	0.	0.
time (sec)	N/A	0.077	0.026	0.051	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	47	24	0	0	0	0
normalized size	1	1.	1.09	0.56	0.	0.	0.	0.
time (sec)	N/A	0.047	0.027	0.036	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.295	0.057	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.57	0.087	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	216	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.286	0.181	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	126	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.04	0.086	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	140	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.131	0.105	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	78	80	0	0	0	0
normalized size	1	1.	0.93	0.95	0.	0.	0.	0.
time (sec)	N/A	0.066	0.026	0.086	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	69	65	0	0	0	0
normalized size	1	1.	1.08	1.02	0.	0.	0.	0.
time (sec)	N/A	0.11	0.068	0.073	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.383	0.061	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	343	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.584	0.333	0.184	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	174	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.441	0.409	0.086	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	225	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.372	0.138	0.103	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	98	119	0	0	0	0
normalized size	1	1.	0.83	1.01	0.	0.	0.	0.
time (sec)	N/A	0.223	0.164	0.086	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	105	81	0	0	0	0
normalized size	1	1.	1.25	0.96	0.	0.	0.	0.
time (sec)	N/A	0.117	0.098	0.076	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.389	0.058	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	285	285	334	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.544	0.673	0.18	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	210	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.661	0.098	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	221	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.434	0.353	0.098	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	118	147	0	0	0	0
normalized size	1	1.	0.8	1.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.295	0.092	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	111	105	0	0	0	0
normalized size	1	1.	0.99	0.94	0.	0.	0.	0.
time (sec)	N/A	0.189	0.181	0.082	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.392	0.06	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	53	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.825	0.713	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	53	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.743	0.547	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	123	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.038	0.55	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.022	0.546	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.397	0.429	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.411	0.435	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.058	0.066	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	1.06	0.058	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.328	0.058	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	1.198	0.062	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.133	0.056	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.737	0.064	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	145	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.146	0.164	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.065	0.079	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.07	0.092	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	38	0	0	0	0
normalized size	1	1.	1.	0.64	0.	0.	0.	0.
time (sec)	N/A	0.084	0.015	0.061	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	45	40	0	0	0	0
normalized size	1	1.	0.92	0.82	0.	0.	0.	0.
time (sec)	N/A	0.047	0.025	0.053	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.276	0.046	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.895	0.053	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	215	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.605	0.402	0.134	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	127	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.432	0.095	0.059	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.225	0.039	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	215	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.862	0.287	0.113	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	129	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.478	0.1	0.052	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	1.034	0.037	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	215	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	1.25	0.432	180.	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	115	0	0	0	0	0
normalized size	1	1.	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.751	0.072	0.057	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	282	0	0	0	0	0
normalized size	1	1.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.406	3.108	0.048	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	196	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.35	0.217	0.115	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	108	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.065	0.059	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.096	0.04	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	290	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	0.379	0.109	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	134	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.109	0.056	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	0.226	0.037	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	340	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.899	1.509	0.111	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	200	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.517	0.71	0.051	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	181	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.273	0.577	0.04	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	346	346	417	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	1.033	1.56	0.109	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	208	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.506	1.138	0.053	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	210	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.447	0.583	0.039	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [30] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	8	0.375
2	A	4	3	1.	8	0.375
3	A	4	3	1.	8	0.375
4	A	3	3	1.	6	0.5
5	A	2	2	1.	4	0.5
6	A	5	5	1.	8	0.625
7	A	4	4	1.	8	0.5
8	A	2	2	1.	8	0.25
9	A	5	5	1.	8	0.625
10	A	3	3	1.	8	0.375
11	A	6	5	1.	8	0.625
12	A	7	5	1.	10	0.5
13	A	6	4	1.	10	0.4
14	A	5	5	1.	10	0.5
15	A	4	4	1.	8	0.5

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	3	1.	6	0.5
17	A	6	6	1.	10	0.6
18	A	7	5	1.	10	0.5
19	A	3	3	1.	10	0.3
20	A	9	7	1.	10	0.7
21	A	5	5	1.	10	0.5
22	A	14	7	1.	10	0.7
23	A	11	5	1.	10	0.5
24	A	9	7	1.	10	0.7
25	A	6	5	1.	8	0.625
26	A	4	3	1.	6	0.5
27	A	7	7	1.	10	0.7
28	A	9	6	1.	10	0.6
29	A	7	7	1.	10	0.7
30	A	14	10	1.	10	1.
31	A	10	9	1.	10	0.9
32	A	23	4	1.	10	0.4
33	A	19	6	1.	10	0.6
34	A	14	4	1.	10	0.4
35	A	11	6	1.	10	0.6
36	A	7	4	1.	8	0.5
37	A	5	3	1.	6	0.5
38	A	8	7	1.	10	0.7
39	A	11	7	1.	10	0.7
40	A	8	8	1.	10	0.8
41	A	19	10	1.	10	1.
42	A	7	3	1.	10	0.3
43	A	6	3	1.	10	0.3
44	A	6	3	1.	10	0.3
45	A	5	3	1.	10	0.3
46	A	5	3	1.	10	0.3
47	A	4	4	1.	8	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	2	2	1.	6	0.333
49	A	0	0	0.	0	0.
50	A	0	0	0.	0	0.
51	A	6	2	1.	10	0.2
52	A	5	2	1.	10	0.2
53	A	5	2	1.	10	0.2
54	A	4	2	1.	10	0.2
55	A	4	2	1.	10	0.2
56	A	2	2	1.	8	0.25
57	A	3	3	1.	6	0.5
58	A	0	0	0.	0	0.
59	A	0	0	0.	0	0.
60	A	14	5	1.	10	0.5
61	A	12	6	1.	10	0.6
62	A	10	6	1.	10	0.6
63	A	7	7	1.	8	0.875
64	A	4	4	1.	6	0.667
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	12	4	1.	10	0.4
68	A	9	4	1.	10	0.4
69	A	10	6	1.	10	0.6
70	A	5	5	1.	8	0.625
71	A	5	4	1.	6	0.667
72	A	0	0	0.	0	0.
73	A	0	0	0.	0	0.
74	A	19	7	1.	12	0.583
75	A	14	7	1.	12	0.583
76	A	14	7	1.	12	0.583
77	A	9	7	1.	10	0.7
78	A	7	6	1.	8	0.75
79	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	41	10	1.	12	0.833
81	A	25	10	1.	12	0.833
82	A	22	10	1.	12	0.833
83	A	11	10	1.	10	1.
84	A	8	7	1.	8	0.875
85	A	0	0	0.	0	0.
86	A	44	10	1.	12	0.833
87	A	27	9	1.	12	0.75
88	A	24	10	1.	12	0.833
89	A	12	9	1.	10	0.9
90	A	9	7	1.	8	0.875
91	A	0	0	0.	0	0.
92	A	18	6	1.	12	0.5
93	A	13	6	1.	12	0.5
94	A	13	6	1.	12	0.5
95	A	8	7	1.	10	0.7
96	A	6	5	1.	8	0.625
97	A	0	0	0.	0	0.
98	A	0	0	0.	0	0.
99	A	17	5	1.	12	0.417
100	A	12	5	1.	12	0.417
101	A	12	5	1.	12	0.417
102	A	6	5	1.	10	0.5
103	A	7	6	1.	8	0.75
104	A	0	0	0.	0	0.
105	A	34	8	1.	12	0.667
106	A	24	9	1.	12	0.75
107	A	22	9	1.	12	0.75
108	A	11	10	1.	10	1.
109	A	8	7	1.	8	0.875
110	A	0	0	0.	0	0.
111	A	32	7	1.	12	0.583

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	21	7	1.	12	0.583
113	A	22	9	1.	12	0.75
114	A	9	8	1.	10	0.8
115	A	9	7	1.	8	0.875
116	A	0	0	0.	0	0.
117	A	0	0	0.	0	0.
118	A	0	0	0.	0	0.
119	A	2	2	1.	10	0.2
120	A	2	2	1.	8	0.25
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	0	0	0.	0	0.
125	A	0	0	0.	0	0.
126	A	0	0	0.	0	0.
127	A	0	0	0.	0	0.
128	A	0	0	0.	0	0.
129	A	12	4	1.	10	0.4
130	A	9	4	1.	10	0.4
131	A	9	4	1.	10	0.4
132	A	6	5	1.	8	0.625
133	A	4	3	1.	6	0.5
134	A	0	0	0.	0	0.
135	A	0	0	0.	0	0.
136	A	14	7	1.	16	0.438
137	A	9	7	1.	14	0.5
138	A	7	6	1.	12	0.5
139	A	22	10	1.	16	0.625
140	A	11	10	1.	14	0.714
141	A	8	7	1.	12	0.583
142	A	24	10	1.	16	0.625
143	A	12	9	1.	14	0.643

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	9	7	1.	12	0.583
145	A	13	6	1.	16	0.375
146	A	8	7	1.	14	0.5
147	A	6	5	1.	12	0.417
148	A	12	5	1.	16	0.312
149	A	6	5	1.	14	0.357
150	A	7	6	1.	12	0.5
151	A	22	9	1.	16	0.562
152	A	11	10	1.	14	0.714
153	A	8	7	1.	12	0.583
154	A	22	9	1.	16	0.562
155	A	9	8	1.	14	0.571
156	A	9	7	1.	12	0.583

Chapter 3

Listing of integrals

3.1 $\int x^4 \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=72

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{25a^5}+\frac{2\left(a^2x^2+1\right)^{3/2}}{15a^5}-\frac{\sqrt{a^2x^2+1}}{5a^5}+\frac{1}{5}x^5\sinh^{-1}(ax)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(5*a^5) + (2*(1 + a^2*x^2)^(3/2))/(15*a^5) - (1 + a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcSinh[a*x])/5
```

Rubi [A] time = 0.0441345, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 266, 43}

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{25a^5}+\frac{2\left(a^2x^2+1\right)^{3/2}}{15a^5}-\frac{\sqrt{a^2x^2+1}}{5a^5}+\frac{1}{5}x^5\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x],x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(5*a^5) + (2*(1 + a^2*x^2)^(3/2))/(15*a^5) - (1 + a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcSinh[a*x])/5
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4 \sinh^{-1}(ax)\,dx &= \frac{1}{5}x^5\sinh^{-1}(ax) - \frac{1}{5}a\int \frac{x^5}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{1}{5}x^5\sinh^{-1}(ax) - \frac{1}{10}a\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\sinh^{-1}(ax) - \frac{1}{10}a\,\mathrm{Subst}\left(\int \left(\frac{1}{a^4\sqrt{1+a^2x}} - \frac{2\sqrt{1+a^2x}}{a^4} + \frac{\left(1+a^2x\right)^{3/2}}{a^4}\right)dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{5a^5} + \frac{2\left(1+a^2x^2\right)^{3/2}}{15a^5} - \frac{\left(1+a^2x^2\right)^{5/2}}{25a^5} + \frac{1}{5}x^5\sinh^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.0347631, size = 50, normalized size = 0.69

$$
\frac{1}{5}x^5\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}\left(3a^4x^4 - 4a^2x^2 + 8\right)}{75a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x],x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4))/(75*a^5) + (x^5*ArcSinh[a*
x])/5
```

Maple [A] time = 0.031, size = 69, normalized size = 1.

$$\frac{1}{a^5}\left(\frac{a^5x^5\mathrm{Arcsinh}\,(ax)}{5}-\frac{a^4x^4}{25}\sqrt{a^2x^2+1}+\frac{4\,a^2x^2}{75}\sqrt{a^2x^2+1}-\frac{8}{75}\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x),x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)-1/25*a^4*x^4*(a^2*x^2+1)^(1/2)+4/75*a^2*x^2
*(a^2*x^2+1)^(1/2)-8/75*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.13708, size = 92, normalized size = 1.28

$$\frac{1}{5}\,x^5\,\mathrm{arsinh}\,(ax)-\frac{1}{75}\left(\frac{3\,\sqrt{a^2x^2+1}x^4}{a^2}-\frac{4\,\sqrt{a^2x^2+1}x^2}{a^4}+\frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x) - 1/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2 +
 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a
```

Fricas [A] time = 1.83797, size = 135, normalized size = 1.88

$$\frac{15\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)-\left(3\,a^4x^4-4\,a^2x^2+8\right)\sqrt{a^2x^2+1}}{75\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1)) - (3*a^4*x^4 - 4*a^2*x^2 + 8)
*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] time = 2.16633, size = 70, normalized size = 0.97

$$\begin{cases} \frac{x^5 \operatorname{asinh}(ax)}{5} - \frac{x^4\sqrt{a^2x^2+1}}{25a} + \frac{4x^2\sqrt{a^2x^2+1}}{75a^3} - \frac{8\sqrt{a^2x^2+1}}{75a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x),x)
```

```
[Out] Piecewise((x**5*asinh(a*x)/5 - x**4*sqrt(a**2*x**2 + 1)/(25*a) + 4*x**2*sqr
t(a**2*x**2 + 1)/(75*a**3) - 8*sqrt(a**2*x**2 + 1)/(75*a**5), Ne(a, 0)), (0
, True))
```

Giac [A] time = 1.30164, size = 90, normalized size = 1.25

$$\frac{1}{5}\,x^5 \log\left(ax + \sqrt{a^2x^2+1}\right) - \frac{3\left(a^2x^2+1\right)^{\frac{5}{2}} - 10\left(a^2x^2+1\right)^{\frac{3}{2}} + 15\sqrt{a^2x^2+1}}{75\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 + 1)) - 1/75*(3*(a^2*x^2 + 1)^(5/2) - 10*(a^
2*x^2 + 1)^(3/2) + 15*sqrt(a^2*x^2 + 1))/a^5
```

3.2 $\int x^3 \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=67

$$-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}-\frac{3\sinh^{-1}(ax)}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)$$

```
[Out] (3*x*Sqrt[1 + a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 + a^2*x^2])/(16*a) - (3*ArcS
inh[a*x])/(32*a^4) + (x^4*ArcSinh[a*x])/4
```

Rubi [A] time = 0.0264488, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 321, 215}

$$-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}-\frac{3\sinh^{-1}(ax)}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 + a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 + a^2*x^2])/(16*a) - (3*ArcS
inh[a*x])/(32*a^4) + (x^4*ArcSinh[a*x])/4
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \sinh^{-1}(ax)\,dx &= \frac{1}{4}x^4\sinh^{-1}(ax) - \frac{1}{4}a\int \frac{x^4}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{16a} + \frac{1}{4}x^4\sinh^{-1}(ax) + \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{16a} \\
&= \frac{3x\sqrt{1+a^2x^2}}{32a^3} - \frac{x^3\sqrt{1+a^2x^2}}{16a} + \frac{1}{4}x^4\sinh^{-1}(ax) - \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{32a^3} \\
&= \frac{3x\sqrt{1+a^2x^2}}{32a^3} - \frac{x^3\sqrt{1+a^2x^2}}{16a} - \frac{3\sinh^{-1}(ax)}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.016575, size = 49, normalized size = 0.73

$$\frac{ax\sqrt{a^2x^2+1}\left(3-2a^2x^2\right)+\left(8a^4x^4-3\right)\sinh^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x],x]
```

```
[Out] (a*x*(3 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + (-3 + 8*a^4*x^4)*ArcSinh[a*x])/(32
*a^4)
```

Maple [A] time = 0.005, size = 58, normalized size = 0.9

$$\frac{1}{a^4}\left(\frac{a^4x^4\mathrm{Arcsinh}\,(ax)}{4} - \frac{a^3x^3}{16}\sqrt{a^2x^2+1} + \frac{3\,ax}{32}\sqrt{a^2x^2+1} - \frac{3\,\mathrm{Arcsinh}\,(ax)}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x),x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)-1/16*a^3*x^3*(a^2*x^2+1)^(1/2)+3/32*a*x*(a^
2*x^2+1)^(1/2)-3/32*arcsinh(a*x))
```

Maxima [A] time = 1.17873, size = 96, normalized size = 1.43

$$\frac{1}{4}\,x^4\,\operatorname{arsinh}\left(ax\right) - \frac{1}{32}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\,\sqrt{a^2x^2+1}x}{a^4} + \frac{3\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x) - 1/32*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 +
 1)*x/a^4 + 3*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4))*a
```

Fricas [A] time = 1.86331, size = 131, normalized size = 1.96

$$\frac{\left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1)) - (2*a^3*x^3 - 3*a*x)*sq
rt(a^2*x^2 + 1))/a^4
```

Sympy [A] time = 1.1598, size = 61, normalized size = 0.91

$$\begin{cases} \frac{x^4\operatorname{asinh}\left(ax\right)}{4} - \frac{x^3\sqrt{a^2x^2+1}}{16a} + \frac{3x\sqrt{a^2x^2+1}}{32a^3} - \frac{3\operatorname{asinh}\left(ax\right)}{32a^4} & \text{for}\; a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x),x)
```

```
[Out] Piecewise((x**4*asinh(a*x)/4 - x**3*sqrt(a**2*x**2 + 1)/(16*a) + 3*x*sqrt(a
**2*x**2 + 1)/(32*a**3) - 3*asinh(a*x)/(32*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.37658, size = 108, normalized size = 1.61

$$\frac{1}{4}\,x^4\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{1}{32}\left(\sqrt{a^2x^2+1}x\left(\frac{2\,x^2}{a^2}-\frac{3}{a^4}\right)-\frac{3\,\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{a^4|a|}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1)) - 1/32*(sqrt(a^2*x^2 + 1)*x*(2*x^2/a^2
 - 3/a^4) - 3*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^4*abs(a)))*a
```

3.3 $\int x^2 \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=52

$$-\frac{\left(a^2x^2+1\right)^{3/2}}{9a^3}+\frac{\sqrt{a^2x^2+1}}{3a^3}+\frac{1}{3}x^3\sinh^{-1}(ax)$$

```
[Out] Sqrt[1 + a^2*x^2]/(3*a^3) - (1 + a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcSinh[a*x]
)/3
```

Rubi [A] time = 0.0345017, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 266, 43}

$$-\frac{\left(a^2x^2+1\right)^{3/2}}{9a^3}+\frac{\sqrt{a^2x^2+1}}{3a^3}+\frac{1}{3}x^3\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x],x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(3*a^3) - (1 + a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcSinh[a*x]
)/3
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh^{-1}(ax)\,dx &= \frac{1}{3}x^3\sinh^{-1}(ax) - \frac{1}{3}a\int \frac{x^3}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{1}{3}x^3\sinh^{-1}(ax) - \frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\sinh^{-1}(ax) - \frac{1}{6}a\,\mathrm{Subst}\left(\int\left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, x^2\right) \\
&= \frac{\sqrt{1+a^2x^2}}{3a^3} - \frac{\left(1+a^2x^2\right)^{3/2}}{9a^3} + \frac{1}{3}x^3\sinh^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.0236022, size = 41, normalized size = 0.79

$$
\frac{1}{9}\left(\frac{\left(2-a^2x^2\right)\sqrt{a^2x^2+1}}{a^3} + 3x^3\sinh^{-1}(ax)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x],x]
```

```
[Out] (((2 - a^2*x^2)*Sqrt[1 + a^2*x^2])/a^3 + 3*x^3*ArcSinh[a*x])/9
```

Maple [A] time = 0.005, size = 50, normalized size = 1.

$$
\frac{1}{a^3}\left(\frac{a^3x^3\operatorname{Arcsinh}(ax)}{3} - \frac{a^2x^2}{9}\sqrt{a^2x^2+1} + \frac{2}{9}\sqrt{a^2x^2+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x),x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)-1/9*a^2*x^2*(a^2*x^2+1)^(1/2)+2/9*(a^2*x^2+
1)^(1/2))
```

Maxima [A] time = 1.16436, size = 65, normalized size = 1.25

$$\frac{1}{3}\,x^3\,\mathrm{arsinh}\left(ax\right) - \frac{1}{9}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x) - 1/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 +
1)/a^4)
```

Fricas [A] time = 1.77979, size = 113, normalized size = 2.17

$$\frac{3\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right) - \sqrt{a^2x^2+1}\left(a^2x^2-2\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*(a^2*x^2 -
2))/a^3
```

Sympy [A] time = 0.550648, size = 48, normalized size = 0.92

$$\begin{cases} \frac{x^3\,\mathrm{asinh}\,(ax)}{3} - \frac{x^2\sqrt{a^2x^2+1}}{9a} + \frac{2\sqrt{a^2x^2+1}}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x),x)
```

```
[Out] Piecewise((x**3*asinh(a*x)/3 - x**2*sqrt(a**2*x**2 + 1)/(9*a) + 2*sqrt(a**2
*x**2 + 1)/(9*a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.31731, size = 70, normalized size = 1.35

$$\frac{1}{3}\,x^{3}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)-\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}-3\,\sqrt{a^{2}x^{2}+1}}{9\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 + 1)) - 1/9*((a^2*x^2 + 1)^(3/2) - 3*sqrt(a^
2*x^2 + 1))/a^3
```

3.4 $\int x \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=44

$$-\frac{x\sqrt{a^2x^2+1}}{4a}+\frac{\sinh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)$$

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(4*a) + ArcSinh[a*x]/(4*a^2) + (x^2*ArcSinh[a*x])/2
```

Rubi [A] time = 0.0156758, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 321, 215}

$$-\frac{x\sqrt{a^2x^2+1}}{4a}+\frac{\sinh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x],x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(4*a) + ArcSinh[a*x]/(4*a^2) + (x^2*ArcSinh[a*x])/2
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh^{-1}(ax)\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax) - \frac{1}{2}a\int\frac{x^2}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}}{4a} + \frac{1}{2}x^2\sinh^{-1}(ax) + \frac{\int\frac{1}{\sqrt{1+a^2x^2}}\,dx}{4a}\\
&= -\frac{x\sqrt{1+a^2x^2}}{4a} + \frac{\sinh^{-1}(ax)}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.009513, size = 40, normalized size = 0.91

$$\frac{\left(2a^2x^2+1\right)\sinh^{-1}(ax) - ax\sqrt{a^2x^2+1}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x],x]
```

```
[Out] (-(a*x*Sqrt[1 + a^2*x^2]) + (1 + 2*a^2*x^2)*ArcSinh[a*x])/(4*a^2)
```

Maple [A] time = 0.003, size = 39, normalized size = 0.9

$$\frac{1}{a^2}\left(\frac{a^2x^2\mathrm{Arcsinh}\,(ax)}{2} - \frac{ax}{4}\sqrt{a^2x^2+1} + \frac{\mathrm{Arcsinh}\,(ax)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x),x)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arcsinh(a*x)-1/4*a*x*(a^2*x^2+1)^(1/2)+1/4*arcsinh(a*x))
```

Maxima [A] time = 1.19328, size = 69, normalized size = 1.57

$$\frac{1}{2}\,x^2\,\mathrm{arsinh}\,(ax) - \frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\mathrm{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a*x) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a^2*x/sqrt(
a^2))/(sqrt(a^2)*a^2))
```

Fricas [A] time = 1.76822, size = 109, normalized size = 2.48

$$-\frac{\sqrt{a^2x^2+1}ax-\left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(a^2*x^2 + 1)*a*x - (2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))
/a^2
```

Sympy [A] time = 0.236765, size = 37, normalized size = 0.84

$$\begin{cases} \frac{x^2\operatorname{asinh}(ax)}{2}-\frac{x\sqrt{a^2x^2+1}}{4a}+\frac{\operatorname{asinh}(ax)}{4a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x),x)
```

```
[Out] Piecewise((x**2*asinh(a*x)/2 - x*sqrt(a**2*x**2 + 1)/(4*a) + asinh(a*x)/(4*
a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.28725, size = 92, normalized size = 2.09

$$\frac{1}{2}\,x^2\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2}+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{a^2|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1)) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 + log
(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^2*abs(a)))
```

3.5 $\int \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=25

$$x\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}}{a}$$

```
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Rubi [A] time = 0.0075172, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5653, 261}

$$x\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x],x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(ax)\,dx &= x\sinh^{-1}(ax) - a\int \frac{x}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{a} + x\sinh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0072242, size = 25, normalized size = 1.

$$x\sinh^{-1}(ax)-\frac{\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x],x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Maple [A] time = 0.002, size = 26, normalized size = 1.

$$\frac{1}{a}\left(ax\mathrm{Arcsinh}\,(ax)-\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x),x)
```

```
[Out] 1/a*(a*x*arcsinh(a*x)-(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.10788, size = 34, normalized size = 1.36

$$\frac{ax\,\mathrm{arsinh}\,(ax)-\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] (a*x*arcsinh(a*x) - sqrt(a^2*x^2 + 1))/a
```

Fricas [A] time = 1.75271, size = 78, normalized size = 3.12

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1))/a
```

Sympy [A] time = 0.152813, size = 20, normalized size = 0.8

$$\begin{cases} x \operatorname{asinh}{(ax)} - \frac{\sqrt{a^2x^2+1}}{a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x),x)
```

```
[Out] Piecewise((x*asinh(a*x) - sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.35222, size = 47, normalized size = 1.88

$$x \log\left(ax + \sqrt{a^2x^2 + 1}\right) - \frac{\sqrt{a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x),x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
```

3.6 $\int \frac{\sinh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\sinh^{-1}(ax)^2+\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -ArcSinh[a*x]^2/2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2, E
^(2*ArcSinh[a*x])]/2
```

Rubi [A] time = 0.0601096, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5659, 3716, 2190, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\sinh^{-1}(ax)^2+\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x,x]
```

```
[Out] -ArcSinh[a*x]^2/2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2, E
^(2*ArcSinh[a*x])]/2
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x}\,dx &= \text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{2}\sinh^{-1}(ax)^2 - 2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(ax)}\right)\\
&= -\frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0030716, size = 43, normalized size = 1.

$$\frac{1}{2}\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x,x]
```

```
[Out] -ArcSinh[a*x]^2/2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2, E
^(2*ArcSinh[a*x])]/2
```

Maple [A] time = 0.148, size = 94, normalized size = 2.2

$$-\frac{(\mathrm{Arcsinh}\,(ax))^2}{2} + \mathrm{Arcsinh}\,(ax)\ln\left(1 + ax + \sqrt{a^2x^2 + 1}\right) + \mathrm{polylog}\left(2, -ax - \sqrt{a^2x^2 + 1}\right) + \mathrm{Arcsinh}\,(ax)\ln\left(1 - ax - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x,x)
```

```
[Out] -1/2*arcsinh(a*x)^2+arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+polylog(2,-a*x
-(a^2*x^2+1)^(1/2))+arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+
(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}\,(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{asinh}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x,x)
```

```
[Out] Integral(asinh(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/x, x)
```

3.7 $\int \frac{\sinh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=27

$$-a\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{x}$$

```
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.0214737, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 266, 63, 208}

$$-a\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^2,x]
```

```
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^2}\,dx &= -\frac{\sinh^{-1}(ax)}{x} + a\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sinh^{-1}(ax)}{x} + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sinh^{-1}(ax)}{x} + \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sinh^{-1}(ax)}{x} - a\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0021349, size = 27, normalized size = 1.

$$-a\tanh^{-1}\left(\sqrt{a^2x^2+1}\right) - \frac{\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^2,x]
```

```
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Maple [A] time = 0.005, size = 30, normalized size = 1.1

$$a\left(-\frac{\mathrm{Arcsinh}\,(ax)}{ax} - \mathrm{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2,x)
```

```
[Out] a*(-arcsinh(a*x)/a/x-arctanh(1/(a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.12306, size = 32, normalized size = 1.19

$$-a\operatorname{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{\operatorname{arsinh}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -a*arcsinh(1/(sqrt(a^2)*abs(x))) - arcsinh(a*x)/x
```

Fricas [B] time = 1.94552, size = 212, normalized size = 7.85

$$-\frac{ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-(x-1)\log\left(ax+\sqrt{a^2x^2+1}\right)-x\log\left(-ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a*x*log(-a*x + sqrt(a^2*x^2 + 1)
- 1) - (x - 1)*log(a*x + sqrt(a^2*x^2 + 1)) - x*log(-a*x + sqrt(a^2*x^2 + 1
)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2,x)
```

```
[Out] Integral(asinh(a*x)/x**2, x)
```

Giac [B] time = 1.38967, size = 76, normalized size = 2.81

$$-\frac{1}{2}\,a\left(\log\left(\sqrt{a^2x^2+1}+1\right)-\log\left(\sqrt{a^2x^2+1}-1\right)\right)-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*a*(log(sqrt(a^2*x^2 + 1) + 1) - log(sqrt(a^2*x^2 + 1) - 1)) - log(a*x
+ sqrt(a^2*x^2 + 1))/x
```

3.8 $\int \frac{\sinh^{-1}(ax)}{x^3} \, dx$

Optimal. Leaf size=33

$$-\frac{a\sqrt{a^2x^2+1}}{2x} - \frac{\sinh^{-1}(ax)}{2x^2}$$

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(2*x) - ArcSinh[a*x]/(2*x^2)
```

Rubi [A] time = 0.0136569, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5661, 264}

$$-\frac{a\sqrt{a^2x^2+1}}{2x} - \frac{\sinh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^3,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(2*x) - ArcSinh[a*x]/(2*x^2)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x^3}\,dx = -\frac{\sinh^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1+a^2x^2}}{2x} - \frac{\sinh^{-1}(ax)}{2x^2}$$

Mathematica [A] time = 0.0063373, size = 28, normalized size = 0.85

$$-\frac{ax\sqrt{a^2x^2+1}+\sinh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^3,x]
```

```
[Out] -(a*x*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/(2*x^2)
```

Maple [A] time = 0.004, size = 37, normalized size = 1.1

$$a^2\left(-\frac{\mathrm{Arcsinh}\,(ax)}{2\,a^2x^2} - \frac{1}{2\,ax}\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^3,x)
```

```
[Out] a^2*(-1/2*arcsinh(a*x)/a^2/x^2-1/2/a/x*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.18445, size = 36, normalized size = 1.09

$$-\frac{\sqrt{a^2x^2+1}\,a}{2\,x} - \frac{\mathrm{arsinh}\,(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a^2*x^2 + 1)*a/x - 1/2*arcsinh(a*x)/x^2
```

Fricas [A] time = 1.94808, size = 88, normalized size = 2.67

$$-\frac{\sqrt{a^2x^2+1}ax+\log\left(ax+\sqrt{a^2x^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(a^2*x^2 + 1)*a*x + log(a*x + sqrt(a^2*x^2 + 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3,x)
```

```
[Out] Integral(asinh(a*x)/x**3, x)
```

Giac [A] time = 1.42686, size = 68, normalized size = 2.06

$$\frac{a|a|}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] a*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1) - 1/2*log(a*x + sqrt(a^2*x^
2 + 1))/x^2
```

3.9 $\int \frac{\sinh^{-1}(ax)}{x^4} \, dx$

Optimal. Leaf size=54

$$-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{3x^3}$$

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(6*x^2) - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

Rubi [A] time = 0.032432, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5661, 266, 51, 63, 208}

$$-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^4,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(6*x^2) - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^4}\,dx &= -\frac{\sinh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sinh^{-1}(ax)}{3x^3}+\frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\sinh^{-1}(ax)}{3x^3}-\frac{1}{12}a^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\sinh^{-1}(ax)}{3x^3}-\frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\sinh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01014, size = 54, normalized size = 1.

$$-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^4,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(6*x^2) - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

Maple [A] time = 0.006, size = 51, normalized size = 0.9

$$a^{3}\left(-\frac{\operatorname{Arcsinh}(ax)}{3a^{3}x^{3}}-\frac{1}{6a^{2}x^{2}}\sqrt{a^{2}x^{2}+1}+\frac{1}{6}\operatorname{Artanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^4,x)
```

```
[Out] a^3*(-1/3*arcsinh(a*x)/a^3/x^3-1/6/a^2/x^2*(a^2*x^2+1)^(1/2)+1/6*arctanh(1/
(a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.11922, size = 61, normalized size = 1.13

$$\frac{1}{6}\left(a^{2}\operatorname{arsinh}\left(\frac{1}{\sqrt{a^{2}}|x|}\right)-\frac{\sqrt{a^{2}x^{2}+1}}{x^{2}}\right)a-\frac{\operatorname{arsinh}(ax)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*arcsinh(1/(sqrt(a^2)*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a - 1/3*arc
sinh(a*x)/x^3
```

Fricas [B] time = 2.03025, size = 273, normalized size = 5.06

$$\frac{a^{3}x^{3}\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)-a^{3}x^{3}\log\left(-ax+\sqrt{a^{2}x^{2}+1}-1\right)+2x^{3}\log\left(-ax+\sqrt{a^{2}x^{2}+1}\right)-\sqrt{a^{2}x^{2}+1}ax+2\left(x^{3}\right.}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^3*x^3*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) + 2*x^3*log(-a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*a
*x + 2*(x^3 - 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**4,x)
```

```
[Out] Integral(asinh(a*x)/x**4, x)
```

Giac [A] time = 1.37784, size = 104, normalized size = 1.93

$$-\frac{1}{12}\,a^3\left(\frac{2\,\sqrt{a^2x^2+1}}{a^2x^2}-\log\left(\sqrt{a^2x^2+1}+1\right)+\log\left(\sqrt{a^2x^2+1}-1\right)\right)-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*a^3*(2*sqrt(a^2*x^2 + 1)/(a^2*x^2) - log(sqrt(a^2*x^2 + 1) + 1) + log
(sqrt(a^2*x^2 + 1) - 1)) - 1/3*log(a*x + sqrt(a^2*x^2 + 1))/x^3
```

3.10 $\int \frac{\sinh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=56

$$\frac{a^3\sqrt{a^2x^2+1}}{6x}-\frac{a\sqrt{a^2x^2+1}}{12x^3}-\frac{\sinh^{-1}(ax)}{4x^4}$$

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(12*x^3) + (a^3*Sqrt[1 + a^2*x^2])/(6*x) - ArcSinh[a
*x]/(4*x^4)
```

Rubi [A] time = 0.0204082, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 271, 264}

$$\frac{a^3\sqrt{a^2x^2+1}}{6x}-\frac{a\sqrt{a^2x^2+1}}{12x^3}-\frac{\sinh^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^5,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(12*x^3) + (a^3*Sqrt[1 + a^2*x^2])/(6*x) - ArcSinh[a
*x]/(4*x^4)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^5}\,dx &= -\frac{\sinh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}}{12x^3}-\frac{\sinh^{-1}(ax)}{4x^4}-\frac{1}{6}a^3\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}}{12x^3}+\frac{a^3\sqrt{1+a^2x^2}}{6x}-\frac{\sinh^{-1}(ax)}{4x^4}
\end{aligned}
$$

Mathematica [A] time = 0.0115542, size = 40, normalized size = 0.71

$$\frac{ax\sqrt{a^2x^2+1}\left(2a^2x^2-1\right)-3\sinh^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^5,x]
```

```
[Out] (a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2) - 3*ArcSinh[a*x])/(12*x^4)
```

Maple [A] time = 0.003, size = 56, normalized size = 1.

$$a^4\left(-\frac{\mathrm{Arcsinh}\,(ax)}{4\,a^4x^4}-\frac{1}{12\,a^3x^3}\sqrt{a^2x^2+1}+\frac{1}{6\,ax}\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^5,x)
```

```
[Out] a^4*(-1/4*arcsinh(a*x)/a^4/x^4-1/12/a^3/x^3*(a^2*x^2+1)^(1/2)+1/6/a/x*(a^2*
x^2+1)^(1/2))
```

Maxima [A] time = 1.17476, size = 66, normalized size = 1.18

$$\frac{1}{12}\left(\frac{2\sqrt{a^2x^2+1}a^2}{x}-\frac{\sqrt{a^2x^2+1}}{x^3}\right)a-\frac{\operatorname{arsinh}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*a - 1/4*arcsinh(a*
x)/x^4
```

Fricas [A] time = 1.83009, size = 109, normalized size = 1.95

$$\frac{\left(2a^3x^3-ax\right)\sqrt{a^2x^2+1}-3\log\left(ax+\sqrt{a^2x^2+1}\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*((2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) - 3*log(a*x + sqrt(a^2*x^2 + 1)))
/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**5,x)
```

```
[Out] Integral(asinh(a*x)/x**5, x)
```

Giac [A] time = 1.35278, size = 104, normalized size = 1.86

$$\frac{\left(3\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)a^3|a|}{3\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^3}-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^3*abs(a)/((x*abs(a) - sqrt(a
^2*x^2 + 1))^2 - 1)^3 - 1/4*log(a*x + sqrt(a^2*x^2 + 1))/x^4
```

3.11 $\int \frac{\sinh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=77

$$\frac{3a^3\sqrt{a^2x^2+1}}{40x^2}-\frac{a\sqrt{a^2x^2+1}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{5x^5}$$

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(20*x^4) + (3*a^3*Sqrt[1 + a^2*x^2])/(40*x^2) - ArcS
inh[a*x]/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 + a^2*x^2]])/40
```

Rubi [A] time = 0.0449983, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5661, 266, 51, 63, 208}

$$\frac{3a^3\sqrt{a^2x^2+1}}{40x^2}-\frac{a\sqrt{a^2x^2+1}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^6,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(20*x^4) + (3*a^3*Sqrt[1 + a^2*x^2])/(40*x^2) - ArcS
inh[a*x]/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 + a^2*x^2]])/40
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^6}\,dx &= -\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{5}a\int \frac{1}{x^5\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}-\frac{\sinh^{-1}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{80}\left(3a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0111742, size = 49, normalized size = 0.64

$$-\frac{1}{5}a^5\sqrt{a^2x^2+1}\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},a^2x^2+1\right)-\frac{\sinh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^6,x]
```

```
[Out] -ArcSinh[a*x]/(5*x^5) - (a^5*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/
2, 1 + a^2*x^2])/5
```

Maple [A] time = 0.006, size = 70, normalized size = 0.9

$$a^5\left(-\frac{\operatorname{Arcsinh}(ax)}{5\,a^5x^5}-\frac{1}{20\,a^4x^4}\sqrt{a^2x^2+1}+\frac{3}{40\,a^2x^2}\sqrt{a^2x^2+1}-\frac{3}{40}\operatorname{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^6,x)
```

```
[Out] a^5*(-1/5*arcsinh(a*x)/a^5/x^5-1/20/a^4/x^4*(a^2*x^2+1)^(1/2)+3/40/a^2/x^2*
(a^2*x^2+1)^(1/2)-3/40*arctanh(1/(a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.16405, size = 88, normalized size = 1.14

$$-\frac{1}{40}\left(3\,a^4\operatorname{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{3\sqrt{a^2x^2+1}a^2}{x^2}+\frac{2\sqrt{a^2x^2+1}}{x^4}\right)a-\frac{\operatorname{arsinh}(ax)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/40*(3*a^4*arcsinh(1/(sqrt(a^2)*abs(x))) - 3*sqrt(a^2*x^2 + 1)*a^2/x^2 +
2*sqrt(a^2*x^2 + 1)/x^4)*a - 1/5*arcsinh(a*x)/x^5
```

Fricas [B] time = 1.95841, size = 302, normalized size = 3.92

$$-\frac{3\,a^5x^5\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-3\,a^5x^5\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-8\,x^5\log\left(-ax+\sqrt{a^2x^2+1}\right)-8\left(x^5-1\right)\log\left(a\right.}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/40*(3*a^5*x^5*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) - 8*x^5*log(-a*x + sqrt(a^2*x^2 + 1)) - 8*(x^5 - 1)*l
og(a*x + sqrt(a^2*x^2 + 1)) - (3*a^3*x^3 - 2*a*x)*sqrt(a^2*x^2 + 1))/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**6,x)
```

```
[Out] Integral(asinh(a*x)/x**6, x)
```

Giac [A] time = 1.42769, size = 128, normalized size = 1.66

$$\frac{1}{80}\,a^5\left(\frac{2\left(3\left(a^2x^2+1\right)^{\frac{3}{2}}-5\sqrt{a^2x^2+1}\right)}{a^4x^4}-3\,\log\left(\sqrt{a^2x^2+1}+1\right)+3\,\log\left(\sqrt{a^2x^2+1}-1\right)\right)-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] 1/80*a^5*(2*(3*(a^2*x^2 + 1)^(3/2) - 5*sqrt(a^2*x^2 + 1))/(a^4*x^4) - 3*log
(sqrt(a^2*x^2 + 1) + 1) + 3*log(sqrt(a^2*x^2 + 1) - 1)) - 1/5*log(a*x + sqr
t(a^2*x^2 + 1))/x^5
```

3.12 $\int x^4 \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=117

$$-\frac{8x^3}{225a^2}-\frac{2x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{25a}+\frac{8x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{75a^3}-\frac{16\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{75a^5}+\frac{16x}{75a^4}+\frac{1}{5}x^5\sinh^{-1}(ax)^2$$

```
[Out] (16*x)/(75*a^4) - (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x])/(75*a^5) + (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(75*a^3) - (
2*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(25*a) + (x^5*ArcSinh[a*x]^2)/5
```

Rubi [A] time = 0.188201, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5758, 5717, 8, 30}

$$-\frac{8x^3}{225a^2}-\frac{2x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{25a}+\frac{8x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{75a^3}-\frac{16\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{75a^5}+\frac{16x}{75a^4}+\frac{1}{5}x^5\sinh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^2,x]
```

```
[Out] (16*x)/(75*a^4) - (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x])/(75*a^5) + (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(75*a^3) - (
2*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(25*a) + (x^5*ArcSinh[a*x]^2)/5
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4 \sinh^{-1}(ax)^2\,dx &= \frac{1}{5}x^5\sinh^{-1}(ax)^2 - \frac{1}{5}(2a)\int \frac{x^5\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^2 + \frac{2\int x^4\,dx}{25} + \frac{8\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{25a} \\
&= \frac{2x^5}{125} + \frac{8x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^2 - \frac{16\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}}{75a^3} \\
&= -\frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax}{25a} \\
&= \frac{16x}{75a^4} - \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x^2}\sinh}{25a}
\end{aligned}$$

Mathematica [A] time = 0.0642595, size = 75, normalized size = 0.64

$$\frac{-\frac{40x^3}{a^2} - \frac{30\sqrt{a^2x^2+1}\left(3a^4x^4-4a^2x^2+8\right)\sinh^{-1}(ax)}{a^5} + \frac{240x}{a^4} + 225x^5\sinh^{-1}(ax)^2 + 18x^5}{1125}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^2,x]
```

```
[Out] ((240*x)/a^4 - (40*x^3)/a^2 + 18*x^5 - (30*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2
 + 3*a^4*x^4)*ArcSinh[a*x])/a^5 + 225*x^5*ArcSinh[a*x]^2)/1125
```

Maple [A] time = 0.131, size = 153, normalized size = 1.3

$$\frac{1}{a^5}\left(\frac{a^3x^3\left(\mathrm{Arcsinh}\left(ax\right)\right)^2\left(a^2x^2+1\right)}{5}-\frac{\left(\mathrm{Arcsinh}\left(ax\right)\right)^2ax\left(a^2x^2+1\right)}{5}+\frac{\left(\mathrm{Arcsinh}\left(ax\right)\right)^2ax}{5}-\frac{2\,\mathrm{Arcsinh}\left(ax\right)a^2x^2}{25}\left(a^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^5*(1/5*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)-1/5*arcsinh(a*x)^2*a*x*(a^2*x
^2+1)+1/5*arcsinh(a*x)^2*a*x-2/25*arcsinh(a*x)*a^2*x^2*(a^2*x^2+1)^(3/2)+14
/75*arcsinh(a*x)*a^2*x^2*(a^2*x^2+1)^(1/2)-16/75*arcsinh(a*x)*(a^2*x^2+1)^(
1/2)+2/125*a*x*(a^2*x^2+1)^2+298/1125*a*x-76/1125*a*x*(a^2*x^2+1))
```

Maxima [A] time = 1.15904, size = 134, normalized size = 1.15

$$\frac{1}{5}\,x^5\,\mathrm{arsinh}\left(ax\right)^2-\frac{2}{75}\left(\frac{3\,\sqrt{a^2x^2+1}x^4}{a^2}-\frac{4\,\sqrt{a^2x^2+1}x^2}{a^4}+\frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a\,\mathrm{arsinh}\left(ax\right)+\frac{2\left(9\,a^4x^5-20\,a^2x^3+120\,x\right)}{1125\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^2 - 2/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x) + 2/1125*(9*a^4*x^5
 - 20*a^2*x^3 + 120*x)/a^4
```

Fricas [A] time = 1.7856, size = 234, normalized size = 2.

$$\frac{225\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^2+18\,a^5x^5-40\,a^3x^3-30\left(3\,a^4x^4-4\,a^2x^2+8\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)+240\,ax}{1125\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^2 + 18*a^5*x^5 - 40*a^3*x^
3 - 30*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1)) + 240*a*x)/a^5
```

Sympy [A] time = 4.13875, size = 114, normalized size = 0.97

$$\begin{cases} \frac{x^5 \operatorname{asinh}^2(ax)}{5} + \frac{2x^5}{125} - \frac{2x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{25a} - \frac{8x^3}{225a^2} + \frac{8x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{75a^3} + \frac{16x}{75a^4} - \frac{16\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{75a^5} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**2/5 + 2*x**5/125 - 2*x**4*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(25*a) - 8*x**3/(225*a**2) + 8*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(75*a**3) + 16*x/(75*a**4) - 16*sqrt(a**2*x**2 + 1)*asinh(a*x)/(75*a**5),
 Ne(a, 0)), (0, True))
```

Giac [A] time = 1.40166, size = 153, normalized size = 1.31

$$\frac{1}{5}\,x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^2+\frac{2}{1125}\,a\left(\frac{9\,a^4x^5-20\,a^2x^3+120\,x}{a^5}-\frac{15\left(3\left(a^2x^2+1\right)^{\frac{5}{2}}-10\left(a^2x^2+1\right)^{\frac{3}{2}}+15\,\sqrt{a^2x^2+1}\right)\log}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2/1125*a*((9*a^4*x^5 - 20*a^2*x^3
+ 120*x)/a^5 - 15*(3*(a^2*x^2 + 1)^(5/2) - 10*(a^2*x^2 + 1)^(3/2) + 15*sqrt
(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))/a^6)
```

3.13 $\int x^3 \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=96

$$-\frac{3x^2}{32a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{16a^3}-\frac{3\sinh^{-1}(ax)^2}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)^2+\frac{x^4}{32}$$

```
[Out] (-3*x^2)/(32*a^2) + x^4/32 + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(16*a^3)
- (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a) - (3*ArcSinh[a*x]^2)/(32*a^4)
+ (x^4*ArcSinh[a*x]^2)/4
```

Rubi [A] time = 0.165311, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5661, 5758, 5675, 30}

$$-\frac{3x^2}{32a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{16a^3}-\frac{3\sinh^{-1}(ax)^2}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)^2+\frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^2,x]
```

```
[Out] (-3*x^2)/(32*a^2) + x^4/32 + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(16*a^3)
- (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a) - (3*ArcSinh[a*x]^2)/(32*a^4)
+ (x^4*ArcSinh[a*x]^2)/4
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3 \sinh^{-1}(ax)^2\,dx &= \frac{1}{4}x^4\sinh^{-1}(ax)^2 - \frac{1}{2}a\int \frac{x^4\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a} + \frac{1}{4}x^4\sinh^{-1}(ax)^2 + \frac{\int x^3\,dx}{8} + \frac{3\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a}\\ &= \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a} + \frac{1}{4}x^4\sinh^{-1}(ax)^2 - \frac{3\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{16a^3} -\\ &= -\frac{3x^2}{32a^2} + \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a} - \frac{3\sinh^{-1}(ax)^2}{32a^4} + \frac{1}{4}x^4\sinh^{-1}\end{aligned}$$

Mathematica [A] time = 0.0444278, size = 72, normalized size = 0.75

$$\frac{a^2x^2\left(a^2x^2-3\right)-2ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)+\left(8a^4x^4-3\right)\sinh^{-1}(ax)^2}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^2,x]
```

```
[Out] (a^2*x^2*(-3 + a^2*x^2) - 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[
a*x] + (-3 + 8*a^4*x^4)*ArcSinh[a*x]^2)/(32*a^4)
```

Maple [A] time = 0.035, size = 118, normalized size = 1.2

$$\frac{1}{a^4}\left(\frac{a^2x^2\left(\operatorname{Arcsinh}\left(ax\right)\right)^2\left(a^2x^2+1\right)}{4}-\frac{\left(\operatorname{Arcsinh}\left(ax\right)\right)^2\left(a^2x^2+1\right)}{4}-\frac{\operatorname{Arcsinh}\left(ax\right)ax}{8}\left(a^2x^2+1\right)^{\frac{3}{2}}+\frac{5\operatorname{Arcsinh}\left(ax\right)ax}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)-1/4*arcsinh(a*x)^2*(a^2*x^2+1
)-1/8*arcsinh(a*x)*a*x*(a^2*x^2+1)^(3/2)+5/16*arcsinh(a*x)*(a^2*x^2+1)^(1/2
)*a*x+5/32*arcsinh(a*x)^2+1/32*a^2*x^2*(a^2*x^2+1)-1/8*a^2*x^2-1/8)
```

Maxima [A] time = 1.269, size = 173, normalized size = 1.8

$$\frac{1}{4}x^4\operatorname{arsinh}\left(ax\right)^2+\frac{1}{32}\left(\frac{x^4}{a^2}-\frac{3x^2}{a^4}+\frac{3\log\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)^2}{a^6}\right)a^2-\frac{1}{16}\left(\frac{2\sqrt{a^2x^2+1}x^3}{a^2}-\frac{3\sqrt{a^2x^2+1}x}{a^4}+\frac{3\operatorname{arsinh}\left(\right.}{\sqrt{a^2}a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x)^2 + 1/32*(x^4/a^2 - 3*x^2/a^4 + 3*log(a^2*x/sqrt(a^2)
+ sqrt(a^2*x^2 + 1))^2/a^6)*a^2 - 1/16*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqr
t(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4))*a*arcsin
h(a*x)
```

Fricas [A] time = 1.82037, size = 205, normalized size = 2.14

$$\frac{a^4x^4-3\,a^2x^2+\left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(a^4*x^4 - 3*a^2*x^2 + (8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^2
- 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] time = 2.28006, size = 90, normalized size = 0.94

$$\begin{cases} \frac{x^4 \operatorname{asinh}^2(ax)}{4} + \frac{x^4}{32} - \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a} - \frac{3x^2}{32a^2} + \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{16a^3} - \frac{3\operatorname{asinh}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**2/4 + x**4/32 - x**3*sqrt(a**2*x**2 + 1)*asinh(
a*x)/(8*a) - 3*x**2/(32*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(16*a**3
) - 3*asinh(a*x)**2/(32*a**4), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^2, x)
```

3.14 $\int x^2 \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=80

$$-\frac{2x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a}+\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^3}-\frac{4x}{9a^2}+\frac{1}{3}x^3\sinh^{-1}(ax)^2+\frac{2x^3}{27}$$

```
[Out] (-4*x)/(9*a^2) + (2*x^3)/27 + (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^3) -
(2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a) + (x^3*ArcSinh[a*x]^2)/3
```

Rubi [A] time = 0.122978, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5758, 5717, 8, 30}

$$-\frac{2x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a}+\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^3}-\frac{4x}{9a^2}+\frac{1}{3}x^3\sinh^{-1}(ax)^2+\frac{2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) + (2*x^3)/27 + (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^3) -
(2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a) + (x^3*ArcSinh[a*x]^2)/3
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh^{-1}(ax)^2\,dx &= \frac{1}{3}x^3\sinh^{-1}(ax)^2 - \frac{1}{3}(2a)\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^2 + \frac{2\int x^2\,dx}{9} + \frac{4\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{9a} \\
&= \frac{2x^3}{27} + \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^2 - \frac{4\int 1\,dx}{9a^2} \\
&= -\frac{4x}{9a^2} + \frac{2x^3}{27} + \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^2
\end{aligned}
$$

Mathematica [A] time = 0.0578393, size = 59, normalized size = 0.74

$$
\frac{1}{27}\left(2x\left(x^2-\frac{6}{a^2}\right) - \frac{6\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^3} + 9x^3\sinh^{-1}(ax)^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^2,x]
```

```
[Out] (2*x*(-6/a^2 + x^2) - (6*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^3
 + 9*x^3*ArcSinh[a*x]^2)/27
```

Maple [A] time = 0.027, size = 92, normalized size = 1.2

$$\frac{1}{a^3}\left(\frac{(\mathrm{Arcsinh}\,(ax))^2\,ax\left(a^2x^2+1\right)}{3}-\frac{(\mathrm{Arcsinh}\,(ax))^2\,ax}{3}-\frac{2\,\mathrm{Arcsinh}\,(ax)\,a^2x^2}{9}\sqrt{a^2x^2+1}+\frac{4\,\mathrm{Arcsinh}\,(ax)}{9}\sqrt{a^2x^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/3*arcsinh(a*x)^2*a*x*(a^2*x^2+1)-1/3*arcsinh(a*x)^2*a*x-2/9*arcsin
h(a*x)*a^2*x^2*(a^2*x^2+1)^(1/2)+4/9*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+2/27*a*
x*(a^2*x^2+1)-14/27*a*x)
```

Maxima [A] time = 1.23514, size = 95, normalized size = 1.19

$$\frac{1}{3}\,x^3\,\mathrm{arsinh}\,(ax)^2-\frac{2}{9}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\mathrm{arsinh}\,(ax)+\frac{2\left(a^2x^3-6\,x\right)}{27\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^2 - 2/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x) + 2/27*(a^2*x^3 - 6*x)/a^2
```

Fricas [A] time = 1.8043, size = 188, normalized size = 2.35

$$\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a^3x^3-6\,\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-12\,ax}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a^3*x^3 - 6*sqrt(a^2*x^2
 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1)) - 12*a*x)/a^3
```

Sympy [A] time = 1.1273, size = 76, normalized size = 0.95

$$\begin{cases} \frac{x^3 \operatorname{asinh}^2(ax)}{3} + \frac{2x^3}{27} - \frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a} - \frac{4x}{9a^2} + \frac{4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**2/3 + 2*x**3/27 - 2*x**2*sqrt(a**2*x**2 + 1)*as
inh(a*x)/(9*a) - 4*x/(9*a**2) + 4*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**3),
Ne(a, 0)), (0, True))
```

Giac [A] time = 1.43483, size = 120, normalized size = 1.5

$$\frac{1}{3}x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^2+\frac{2}{27}a\left(\frac{a^2x^3-6x}{a^3}-\frac{3\left(\left(a^2x^2+1\right)^{\frac{3}{2}}-3\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2/27*a*((a^2*x^3 - 6*x)/a^3 - 3*((
a^2*x^2 + 1)^(3/2) - 3*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))/a^4)
```

3.15 $\int x \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=59

$$-\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^2+\frac{x^2}{4}$$

```
[Out] x^2/4 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + ArcSinh[a*x]^2/(4*a^2) +
 (x^2*ArcSinh[a*x]^2)/2
```

Rubi [A] time = 0.0918002, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5758, 5675, 30}

$$-\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^2+\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^2,x]
```

```
[Out] x^2/4 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + ArcSinh[a*x]^2/(4*a^2) +
 (x^2*ArcSinh[a*x]^2)/2
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}(ax)^2\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax)^2 - a\int\frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a} + \frac{1}{2}x^2\sinh^{-1}(ax)^2 + \frac{\int x\,dx}{2} + \frac{\int\frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a}\\
&= \frac{x^2}{4} - \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a} + \frac{\sinh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^2
\end{aligned}$$

Mathematica [A] time = 0.0282382, size = 53, normalized size = 0.9

$$\frac{a^2x^2 - 2ax\sqrt{a^2x^2+1}\sinh^{-1}(ax) + \left(2a^2x^2+1\right)\sinh^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^2,x]
```

```
[Out] (a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (1 + 2*a^2*x^2)*ArcSinh[a
*x]^2)/(4*a^2)
```

Maple [A] time = 0.026, size = 59, normalized size = 1.

$$\frac{1}{a^2}\left(\frac{(\mathrm{Arcsinh}\,(ax))^2\left(a^2x^2+1\right)}{2} - \frac{\mathrm{Arcsinh}\,(ax)\,ax}{2}\sqrt{a^2x^2+1} - \frac{(\mathrm{Arcsinh}\,(ax))^2}{4} + \frac{a^2x^2}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^2*(1/2*arcsinh(a*x)^2*(a^2*x^2+1)-1/2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*
x-1/4*arcsinh(a*x)^2+1/4*a^2*x^2+1/4)
```

Maxima [B] time = 1.12721, size = 135, normalized size = 2.29

$$\frac{1}{2}x^2 \operatorname{arsinh}(ax)^2 + \frac{1}{4}a^2\left(\frac{x^2}{a^2} - \frac{\log\left(\frac{a^2x}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)^2}{a^4}\right) - \frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a*x)^2 + 1/4*a^2*(x^2/a^2 - log(a^2*x/sqrt(a^2) + sqrt(a^2*
x^2 + 1))^2/a^4) - 1/2*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a^2*x/sqrt(a^2)
)/(sqrt(a^2)*a^2))*arcsinh(a*x)
```

Fricas [A] time = 2.00562, size = 166, normalized size = 2.81

$$\frac{a^2x^2 - 2\sqrt{a^2x^2+1}ax\log\left(ax + \sqrt{a^2x^2+1}\right) + \left(2a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + (2*a^
2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^2
```

Sympy [A] time = 0.541131, size = 51, normalized size = 0.86

$$\begin{cases} \frac{x^2\operatorname{asinh}^2(ax)}{2} + \frac{x^2}{4} - \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a} + \frac{\operatorname{asinh}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**2/2 + x**2/4 - x*sqrt(a**2*x**2 + 1)*asinh(a*x)
/(2*a) + asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^2, x)
```

3.16 $\int \sinh^{-1}(ax)^2\,dx$

Optimal. Leaf size=34

$$-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2x$$

```
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Rubi [A] time = 0.0451432, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5653, 5717, 8}

$$-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}(ax)^2\,dx &= x\sinh^{-1}(ax)^2-(2a)\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2\int 1\,dx\\
&= 2x-\frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2
\end{aligned}
$$

Mathematica [A] time = 0.0135055, size = 34, normalized size = 1.

$$
-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2x
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Maple [A] time = 0.023, size = 36, normalized size = 1.1

$$
\frac{1}{a}\left((\mathrm{Arcsinh}\,(ax))^2\,ax-2\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}+2\,ax\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2,x)
```

```
[Out] 1/a*(arcsinh(a*x)^2*a*x-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+2*a*x)
```

Maxima [A] time = 1.18164, size = 43, normalized size = 1.26

$$
x\,\mathrm{arsinh}\,(ax)^2+2\,x-\frac{2\,\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^2 + 2*x - 2*sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a
```

Fricas [A] time = 2.10122, size = 134, normalized size = 3.94

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,ax-2\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*x - 2*sqrt(a^2*x^2 + 1)*log(a*x +
 sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] time = 0.218243, size = 32, normalized size = 0.94

$$\begin{cases} x\operatorname{asinh}^2(ax)+2x-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2,x)
```

```
[Out] Piecewise((x*asinh(a*x)**2 + 2*x - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a
, 0)), (0, True))
```

Giac [A] time = 1.45534, size = 84, normalized size = 2.47

$$x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + s
qrt(a^2*x^2 + 1))/a^2)
```

3.17 $\int \frac{\sinh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=60

$$\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{3}\sinh^{-1}(ax)^3+\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -ArcSinh[a*x]^3/3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[a*
x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Rubi [A] time = 0.0967535, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5659, 3716, 2190, 2531, 2282, 6589}

$$\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{3}\sinh^{-1}(ax)^3+\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x,x]
```

```
[Out] -ArcSinh[a*x]^3/3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[a*
x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x}\,dx &= \text{Subst}\left(\int x^2\coth(x)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) - 2\,\text{Subst}\left(\int x\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \mathbf{Li}_2\left(\right.\right.\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\,\text{Subst}\left(\int \mathbf{Li}\right.\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\mathbf{Li}_3\left(e^{2\sinh^{-1}(a\right.
\end{aligned}
$$

Mathematica [A] time = 0.0055132, size = 60, normalized size = 1.

$$\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{3}\sinh^{-1}(ax)^3+\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/x,x]
```

```
[Out] -ArcSinh[a*x]^3/3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[a*
x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Maple [A] time = 0.036, size = 151, normalized size = 2.5

$$-\frac{(\text{Arcsinh}\,(ax))^3}{3}+(\text{Arcsinh}\,(ax))^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+2\,\text{Arcsinh}\,(ax)\,\text{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right)-2\,\text{polyl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x,x)
```

```
[Out] -1/3*arcsinh(a*x)^3+arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*arcsinh(a*
x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+ar
csinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*
x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}^2\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x,x)
```

```
[Out] Integral(asinh(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\,(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x, x)
```

3.18 $\int \frac{\sinh^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=50

$$-2a\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2a\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sinh^{-1}(ax)^2}{x}-4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -(ArcSinh[a*x]^2/x) - 4*a*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - 2*a*PolyLo
g[2, -E^ArcSinh[a*x]] + 2*a*PolyLog[2, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.0997795, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5760, 4182, 2279, 2391}

$$-2a\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2a\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sinh^{-1}(ax)^2}{x}-4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^2/x) - 4*a*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - 2*a*PolyLo
g[2, -E^ArcSinh[a*x]] + 2*a*PolyLog[2, E^ArcSinh[a*x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx &= -\frac{\sinh^{-1}(ax)^2}{x} + (2a)\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sinh^{-1}(ax)^2}{x} + (2a)\,\text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right) \\
&= -\frac{\sinh^{-1}(ax)^2}{x} - 4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (2a)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + \\
&= -\frac{\sinh^{-1}(ax)^2}{x} - 4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (2a)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right) + (2 \\
&= -\frac{\sinh^{-1}(ax)^2}{x} - 4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2a\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2a\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A] time = 0.191734, size = 75, normalized size = 1.5

$$a\left(2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) - 2\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right) - \sinh^{-1}(ax)\left(\frac{\sinh^{-1}(ax)}{ax} - 2\log\left(1-e^{-\sinh^{-1}(ax)}\right) + 2\log\left(e\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/x^2,x]
```

```
[Out] a*(-(ArcSinh[a*x]*(ArcSinh[a*x]/(a*x) - 2*Log[1 - E^(-ArcSinh[a*x])] + 2*Lo
g[1 + E^(-ArcSinh[a*x])])) + 2*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*PolyLog[2
```

```
, E^(-ArcSinh[a*x])])
```

Maple [A] time = 0.049, size = 107, normalized size = 2.1

$$-\frac{(\mathrm{Arcsinh}\,(ax))^2}{x}-2\,a\mathrm{Arcsinh}\,(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)-2\,a\mathrm{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right)+2\,a\mathrm{Arcsinh}\,(ax)\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^2,x)
```

```
[Out] -arcsinh(a*x)^2/x-2*a*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-2*a*polylog(
2,-a*x-(a^2*x^2+1)^(1/2))+2*a*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*
polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{x}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^3x^4+ax^2+\left(a^2x^3+x\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*
sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}\,(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**2,x)
```

```
[Out] Integral(asinh(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x^2, x)
```

3.19 $\int \frac{\sinh^{-1}(ax)^2}{x^3} \, dx$

Optimal. Leaf size=43

$$-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x} + a^2\log(x) - \frac{\sinh^{-1}(ax)^2}{2x^2}$$

```
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

Rubi [A] time = 0.0804655, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5661, 5723, 29}

$$-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x} + a^2\log(x) - \frac{\sinh^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^3,x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^2}{x^3}\,dx &= -\frac{\sinh^{-1}(ax)^2}{2x^2}+a\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x}-\frac{\sinh^{-1}(ax)^2}{2x^2}+a^2\int \frac{1}{x}\,dx\\ &= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x}-\frac{\sinh^{-1}(ax)^2}{2x^2}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0310021, size = 43, normalized size = 1.

$$-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}+a^2\log(x)-\frac{\sinh^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/x^3,x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

Maple [A] time = 0.076, size = 67, normalized size = 1.6

$$-a^2\mathrm{Arcsinh}\,(ax)-\frac{a\mathrm{Arcsinh}\,(ax)}{x}\sqrt{a^2x^2+1}-\frac{(\mathrm{Arcsinh}\,(ax))^2}{2\,x^2}+a^2\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^3,x)
```

```
[Out] -a^2*arcsinh(a*x)-a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x-1/2*arcsinh(a*x)^2/x^2
+a^2*ln((a*x+(a^2*x^2+1)^(1/2))^2-1)
```

Maxima [A] time = 1.1918, size = 53, normalized size = 1.23

$$a^2 \log(x) - \frac{\sqrt{a^2x^2 + 1}a \operatorname{arsinh}(ax)}{x} - \frac{\operatorname{arsinh}(ax)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - sqrt(a^2*x^2 + 1)*a*arcsinh(a*x)/x - 1/2*arcsinh(a*x)^2/x^2
```

Fricas [A] time = 2.17422, size = 157, normalized size = 3.65

$$\frac{2a^2x^2 \log(x) - 2\sqrt{a^2x^2 + 1}ax \log\left(ax + \sqrt{a^2x^2 + 1}\right) - \log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*log(x) - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)
) - log(a*x + sqrt(a^2*x^2 + 1))^2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**3,x)
```

```
[Out] Integral(asinh(a*x)**2/x**3, x)
```

Giac [B] time = 1.48154, size = 149, normalized size = 3.47

$$-\left(a\left(\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}-\frac{|a|\log\left(|x|\right)}{a^2}\right)|a|-\frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}\right)a-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -(a*(log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - abs(a)*log(abs(x))/a^2)*ab
s(a) - 2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1)
)^2 - 1))*a - 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/x^2
```

3.20 $\int \frac{\sinh^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=99

$$\frac{1}{3}a^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{3}a^3\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3x^2}-\frac{a^2}{3x}+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right.$$

```
[Out] -a^2/(3*x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*x^2) - ArcSinh[a*x]^2/(3
*x^3) + (2*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]])/3 + (a^3*PolyLog[2, -E
^ArcSinh[a*x]])/3 - (a^3*PolyLog[2, E^ArcSinh[a*x]])/3
```

Rubi [A] time = 0.162589, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5661, 5747, 5760, 4182, 2279, 2391, 30}

$$\frac{1}{3}a^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{3}a^3\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3x^2}-\frac{a^2}{3x}+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^4,x]
```


```
[Out] -a^2/(3*x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*x^2) - ArcSinh[a*x]^2/(3
*x^3) + (2*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]])/3 + (a^3*PolyLog[2, -E
^ArcSinh[a*x]])/3 - (a^3*PolyLog[2, E^ArcSinh[a*x]])/3
```


Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x^4}\,dx = -\frac{\sinh^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a)\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2} - \frac{\sinh^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^2\int \frac{1}{x^2}\,dx - \frac{1}{3}a^3\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a^2}{3x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2} - \frac{\sinh^{-1}(ax)^2}{3x^3} - \frac{1}{3}a^3\,\text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{a^2}{3x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2} - \frac{\sinh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{3}a^3\,\text{Subst}\,($$
$$= -\frac{a^2}{3x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2} - \frac{\sinh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{3}a^3\,\text{Subst}\,($$
$$= -\frac{a^2}{3x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2} - \frac{\sinh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{3}a^3\text{Li}_2\,(-e^{\text{s}}$$

Mathematica [A] time = 0.489779, size = 125, normalized size = 1.26

$$-\frac{a^3x^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) - a^3x^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right) + a^2x^2 + ax\sqrt{a^2x^2+1}\sinh^{-1}(ax) + a^3x^3\sinh^{-1}(ax)\,\text{lo}}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/x^4,x]
```

```
[Out] -(a^2*x^2 + a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2 + a^3*x^3*A
rcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] - a^3*x^3*ArcSinh[a*x]*Log[1 + E^(-A
rcSinh[a*x])] + a^3*x^3*PolyLog[2, -E^(-ArcSinh[a*x])] - a^3*x^3*PolyLog[2,
 E^(-ArcSinh[a*x])])/(3*x^3)
```

Maple [A] time = 0.092, size = 144, normalized size = 1.5

$$-\frac{a\text{Arcsinh}\,(ax)}{3\,x^2}\sqrt{a^2x^2+1} - \frac{a^2}{3\,x} - \frac{(\text{Arcsinh}\,(ax))^2}{3\,x^3} + \frac{a^3\text{Arcsinh}\,(ax)}{3}\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + \frac{a^3}{3}\text{polylog}\,(2,-ax-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^4,x)
```

```
[Out] -1/3*a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x^2-1/3*a^2/x-1/3*arcsinh(a*x)^2/x^3+
1/3*a^3*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/3*a^3*polylog(2,-a*x-(a^
2*x^2+1)^(1/2))-1/3*a^3*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/3*a^3*po
lylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{3\,x^3}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{3\left(a^3x^6+ax^4+\left(a^2x^5+x^3\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^2/x^3 + integrate(2/3*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^6 + a*x^4 + (a^2*x
^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{arsinh}(ax)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**4,x)
```

```
[Out] Integral(asinh(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x^4, x)
```

3.21 $\int \frac{\sinh^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=85

$$-\frac{a^2}{12x^2}+\frac{a^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3x}-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{6x^3}-\frac{1}{3}a^4\log(x)-\frac{\sinh^{-1}(ax)^2}{4x^4}$$

```
[Out] -a^2/(12*x^2) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(6*x^3) + (a^3*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x])/(3*x) - ArcSinh[a*x]^2/(4*x^4) - (a^4*Log[x])/3
```

Rubi [A] time = 0.137922, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5747, 5723, 29, 30}

$$-\frac{a^2}{12x^2}+\frac{a^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3x}-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{6x^3}-\frac{1}{3}a^4\log(x)-\frac{\sinh^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^5,x]
```

```
[Out] -a^2/(12*x^2) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(6*x^3) + (a^3*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x])/(3*x) - ArcSinh[a*x]^2/(4*x^4) - (a^4*Log[x])/3
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^5}\,dx &= -\frac{\sinh^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{\sinh^{-1}(ax)}{x^4\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{6x^3}-\frac{\sinh^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^2\int \frac{1}{x^3}\,dx-\frac{1}{3}a^3\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a^2}{12x^2}-\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{6x^3}+\frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x}-\frac{\sinh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\int \frac{1}{x}\,dx\\
&= -\frac{a^2}{12x^2}-\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{6x^3}+\frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x}-\frac{\sinh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0592826, size = 64, normalized size = 0.75

$$
-\frac{a^2x^2+4a^4x^4\log(x)-2ax\sqrt{a^2x^2+1}\left(2a^2x^2-1\right)\sinh^{-1}(ax)+3\sinh^{-1}(ax)^2}{12x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/x^5,x]
```

```
[Out] -(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2)*ArcSinh[a*x] + 3*ArcSi
nh[a*x]^2 + 4*a^4*x^4*Log[x])/(12*x^4)
```

Maple [A] time = 0.106, size = 99, normalized size = 1.2

$$\frac{a^4\mathrm{Arcsinh}\,(ax)}{3}+\frac{a^3\mathrm{Arcsinh}\,(ax)}{3\,x}\sqrt{a^2x^2+1}-\frac{a\mathrm{Arcsinh}\,(ax)}{6\,x^3}\sqrt{a^2x^2+1}-\frac{a^2}{12\,x^2}-\frac{(\mathrm{Arcsinh}\,(ax))^2}{4\,x^4}-\frac{a^4}{3}\ln\left(\left(ax+\sqrt{a^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^5,x)
```

```
[Out] 1/3*a^4*arcsinh(a*x)+1/3*a^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x-1/6*a*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)/x^3-1/12*a^2/x^2-1/4*arcsinh(a*x)^2/x^4-1/3*a^4*ln(
(a*x+(a^2*x^2+1)^(1/2))^2-1)
```

Maxima [A] time = 1.22541, size = 96, normalized size = 1.13

$$-\frac{1}{12}\left(4\,a^2\log\left(x\right)+\frac{1}{x^2}\right)a^2+\frac{1}{6}\left(\frac{2\,\sqrt{a^2x^2+1}a^2}{x}-\frac{\sqrt{a^2x^2+1}}{x^3}\right)a\,\mathrm{arsinh}\,(ax)-\frac{\mathrm{arsinh}\,(ax)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^2*log(x) + 1/x^2)*a^2 + 1/6*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^
2*x^2 + 1)/x^3)*a*arcsinh(a*x) - 1/4*arcsinh(a*x)^2/x^4
```

Fricas [A] time = 2.16209, size = 194, normalized size = 2.28

$$-\frac{4\,a^4x^4\log\left(x\right)+a^2x^2-2\left(2\,a^3x^3-ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)+3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^4*x^4*log(x) + a^2*x^2 - 2*(2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1)*l
og(a*x + sqrt(a^2*x^2 + 1)) + 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**5,x)
```

```
[Out] Integral(asinh(a*x)**2/x**5, x)
```

Giac [B] time = 1.59414, size = 200, normalized size = 2.35

$$-\frac{1}{12}\left(2\,a^3\log\left(x^2\right)-4\,a^3\log\left(-x|a|+\sqrt{a^2x^2+1}\right)-\frac{8\left(3\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)a^2|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^3}-\frac{2\,a^3x^2-a}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(2*a^3*log(x^2) - 4*a^3*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - 8*(3*(x*
abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1))/
((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^3 - (2*a^3*x^2 - a)/x^2)*a - 1/4*log
(a*x + sqrt(a^2*x^2 + 1))^2/x^4
```

3.22 $\int x^4 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=195

$$-\frac{6\left(a^2x^2+1\right)^{5/2}}{625a^5}+\frac{76\left(a^2x^2+1\right)^{3/2}}{1125a^5}-\frac{298\sqrt{a^2x^2+1}}{375a^5}-\frac{3x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{25a}-\frac{8x^3\sinh^{-1}(ax)}{75a^2}+\frac{4x^2\sqrt{a^2x^2+1}\sinh}{25a^3}$$

```
[Out] (-298*Sqrt[1 + a^2*x^2])/(375*a^5) + (76*(1 + a^2*x^2)^(3/2))/(1125*a^5) -
(6*(1 + a^2*x^2)^(5/2))/(625*a^5) + (16*x*ArcSinh[a*x])/(25*a^4) - (8*x^3*A
rcSinh[a*x])/(75*a^2) + (6*x^5*ArcSinh[a*x])/125 - (8*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]^2)/(25*a^5) + (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a^3) -
 (3*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a) + (x^5*ArcSinh[a*x]^3)/5
```

Rubi [A] time = 0.36596, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5661, 5758, 5717, 5653, 261, 266, 43}

$$-\frac{6\left(a^2x^2+1\right)^{5/2}}{625a^5}+\frac{76\left(a^2x^2+1\right)^{3/2}}{1125a^5}-\frac{298\sqrt{a^2x^2+1}}{375a^5}-\frac{3x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{25a}-\frac{8x^3\sinh^{-1}(ax)}{75a^2}+\frac{4x^2\sqrt{a^2x^2+1}\sinh}{25a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^3,x]
```


```
[Out] (-298*Sqrt[1 + a^2*x^2])/(375*a^5) + (76*(1 + a^2*x^2)^(3/2))/(1125*a^5) -
(6*(1 + a^2*x^2)^(5/2))/(625*a^5) + (16*x*ArcSinh[a*x])/(25*a^4) - (8*x^3*A
rcSinh[a*x])/(75*a^2) + (6*x^5*ArcSinh[a*x])/125 - (8*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]^2)/(25*a^5) + (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a^3) -
 (3*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a) + (x^5*ArcSinh[a*x]^3)/5
```


Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^3\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^3 - \frac{1}{5}(3a)\int \frac{x^5\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^3 + \frac{6}{25}\int x^4\sinh^{-1}(ax)\,dx + \frac{12\int\frac{x^3\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{25a}$$

$$= \frac{6}{125}x^5\sinh^{-1}(ax) + \frac{4x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^3 -$$

$$= -\frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5\sinh^{-1}(ax) - \frac{8\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{25a^5} + \frac{4x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{25a^3} -$$

$$= \frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5\sinh^{-1}(ax) - \frac{8\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{25a^5} +$$

$$= -\frac{86\sqrt{1+a^2x^2}}{125a^5} + \frac{4\left(1+a^2x^2\right)^{3/2}}{125a^5} - \frac{6\left(1+a^2x^2\right)^{5/2}}{625a^5} + \frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5$$

$$= -\frac{298\sqrt{1+a^2x^2}}{375a^5} + \frac{76\left(1+a^2x^2\right)^{3/2}}{1125a^5} - \frac{6\left(1+a^2x^2\right)^{5/2}}{625a^5} + \frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5$$

Mathematica [A] time = 0.0689866, size = 120, normalized size = 0.62

$$\frac{-2\sqrt{a^2x^2+1}\left(27a^4x^4-136a^2x^2+2072\right)+1125a^5x^5\sinh^{-1}(ax)^3+30ax\left(9a^4x^4-20a^2x^2+120\right)\sinh^{-1}(ax)-225\sqrt{a^2x^2}}{5625a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^3,x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(2072 - 136*a^2*x^2 + 27*a^4*x^4) + 30*a*x*(120 - 20*
a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 +
3*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a^5*x^5*ArcSinh[a*x]^3)/(5625*a^5)
```

Maple [A] time = 0.04, size = 222, normalized size = 1.1

$$\frac{1}{a^5}\left(\frac{a^3x^3\left(\mathrm{Arcsinh}\,(ax)\right)^3\left(a^2x^2+1\right)}{5} - \frac{\left(\mathrm{Arcsinh}\,(ax)\right)^3 ax\left(a^2x^2+1\right)}{5} + \frac{\left(\mathrm{Arcsinh}\,(ax)\right)^3 ax}{5} - \frac{3\left(\mathrm{Arcsinh}\,(ax)\right)^2 a^2x^2}{25}\left(a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^5*(1/5*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)-1/5*arcsinh(a*x)^3*a*x*(a^2*x
^2+1)+1/5*arcsinh(a*x)^3*a*x-3/25*arcsinh(a*x)^2*a^2*x^2*(a^2*x^2+1)^(3/2)+
7/25*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-8/25*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)+6/125*arcsinh(a*x)*a*x*(a^2*x^2+1)^2+298/375*a*x*arcsinh(a*x)-76/3
75*arcsinh(a*x)*a*x*(a^2*x^2+1)-6/625*a^2*x^2*(a^2*x^2+1)^(3/2)+326/5625*a^
2*x^2*(a^2*x^2+1)^(1/2)-4144/5625*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.0839, size = 223, normalized size = 1.14

$$\frac{1}{5}\,x^5\,\operatorname{arsinh}\left(ax\right)^3-\frac{1}{25}\left(\frac{3\,\sqrt{a^2x^2+1}x^4}{a^2}-\frac{4\,\sqrt{a^2x^2+1}x^2}{a^4}+\frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a\,\operatorname{arsinh}\left(ax\right)^2-\frac{2}{5625}\,a\left(\frac{27\,\sqrt{a^2x^2+1}a^2x^4-1\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^3 - 1/25*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^2 - 2/5625*a*((27*s
qrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(a^2*x^2 +
1)/a^2)/a^4 - 15*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)/a^5)
```

Fricas [A] time = 2.16287, size = 359, normalized size = 1.84

$$\frac{1125\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\left(3\,a^4x^4-4\,a^2x^2+8\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+30\left(9\,a^5x^5-20\,a^3x^3+\cdots\right.}{5625\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(1125*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^3 - 225*(3*a^4*x^4 - 4*a^
2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(9*a^5*x^5
 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(27*a^4*x^4 - 136
```

```
*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] time = 7.48059, size = 196, normalized size = 1.01

$$\begin{cases} \frac{x^5\operatorname{asinh}^3(ax)}{5} + \frac{6x^5\operatorname{asinh}(ax)}{125} - \frac{3x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25a} - \frac{6x^4\sqrt{a^2x^2+1}}{625a} - \frac{8x^3\operatorname{asinh}(ax)}{75a^2} + \frac{4x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25a^3} + \frac{272x^2\sqrt{a^2x^2+1}}{5625a^3} + \frac{16x\operatorname{asin}}{25a} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**3/5 + 6*x**5*asinh(a*x)/125 - 3*x**4*sqrt(a**2*
x**2 + 1)*asinh(a*x)**2/(25*a) - 6*x**4*sqrt(a**2*x**2 + 1)/(625*a) - 8*x**
3*asinh(a*x)/(75*a**2) + 4*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(25*a**3)
 + 272*x**2*sqrt(a**2*x**2 + 1)/(5625*a**3) + 16*x*asinh(a*x)/(25*a**4) - 8
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(25*a**5) - 4144*sqrt(a**2*x**2 + 1)/(56
25*a**5), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.59894, size = 243, normalized size = 1.25

$$\frac{1}{5}x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - \frac{1}{5625}a\left(\frac{225\left(3\left(a^2x^2+1\right)^{\frac{5}{2}} - 10\left(a^2x^2+1\right)^{\frac{3}{2}} + 15\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^6} - \frac{2\left(1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^3 - 1/5625*a*(225*(3*(a^2*x^2 + 1)^(5/
2) - 10*(a^2*x^2 + 1)^(3/2) + 15*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))^2/a^6 - 2*(15*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*log(a*x + sqrt(a^2*x^2
 + 1)) - (27*(a^2*x^2 + 1)^(5/2) - 190*(a^2*x^2 + 1)^(3/2) + 2235*sqrt(a^2*
x^2 + 1))/a)/a^5)
```

3.23 $\int x^3 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=163

$$-\frac{3x^3\sqrt{a^2x^2+1}}{128a}+\frac{45x\sqrt{a^2x^2+1}}{256a^3}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{16a}-\frac{9x^2\sinh^{-1}(ax)}{32a^2}+\frac{9x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{32a^3}-\frac{3\sinh^{-1}}{32a}$$

```
[Out] (45*x*Sqrt[1 + a^2*x^2])/(256*a^3) - (3*x^3*Sqrt[1 + a^2*x^2])/(128*a) - (4
5*ArcSinh[a*x])/(256*a^4) - (9*x^2*ArcSinh[a*x])/(32*a^2) + (3*x^4*ArcSinh[
a*x])/32 + (9*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^2)/(16*a) - (3*ArcSinh[a*x]^3)/(32*a^4) + (x^4*ArcS
inh[a*x]^3)/4
```

Rubi [A] time = 0.298368, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5758, 5675, 321, 215}

$$-\frac{3x^3\sqrt{a^2x^2+1}}{128a}+\frac{45x\sqrt{a^2x^2+1}}{256a^3}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{16a}-\frac{9x^2\sinh^{-1}(ax)}{32a^2}+\frac{9x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{32a^3}-\frac{3\sinh^{-1}}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^3,x]
```

```
[Out] (45*x*Sqrt[1 + a^2*x^2])/(256*a^3) - (3*x^3*Sqrt[1 + a^2*x^2])/(128*a) - (4
5*ArcSinh[a*x])/(256*a^4) - (9*x^2*ArcSinh[a*x])/(32*a^2) + (3*x^4*ArcSinh[
a*x])/32 + (9*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^2)/(16*a) - (3*ArcSinh[a*x]^3)/(32*a^4) + (x^4*ArcS
inh[a*x]^3)/4
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^3\,dx = \frac{1}{4}x^4\sinh^{-1}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^4\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\sinh^{-1}(ax)^3 + \frac{3}{8}\int x^3\sinh^{-1}(ax)\,dx + \frac{9\int \frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{16a}$$

$$= \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\sinh^{-1}(ax)^3 - \frac{9}{}$$

$$= -\frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{9x^2\sinh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{16}$$

$$= \frac{45x\sqrt{1+a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{9x^2\sinh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{32a^3}$$

$$= \frac{45x\sqrt{1+a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{45\sinh^{-1}(ax)}{256a^4} - \frac{9x^2\sinh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a}}{}$$

Mathematica [A] time = 0.0690697, size = 110, normalized size = 0.67

$$\frac{3ax\left(15-2a^2x^2\right)\sqrt{a^2x^2+1}+8\left(8a^4x^4-3\right)\sinh^{-1}(ax)^3-24ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^2+3\left(8a^4x^4-24a^2x^2\right.}{256a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^3,x]
```

```
[Out] (3*a*x*(15 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4
)*ArcSinh[a*x] - 24*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 +
 8*(-3 + 8*a^4*x^4)*ArcSinh[a*x]^3)/(256*a^4)
```

Maple [A] time = 0.034, size = 168, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{a^2x^2\left(\mathrm{Arcsinh}\,(ax)\right)^3\left(a^2x^2+1\right)}{4}-\frac{\left(\mathrm{Arcsinh}\,(ax)\right)^3\left(a^2x^2+1\right)}{4}-\frac{3\,\left(\mathrm{Arcsinh}\,(ax)\right)^2ax}{16}\left(a^2x^2+1\right)^{\frac{3}{2}}+\frac{15\,\left(\mathrm{Arcsinh}\,(\right.}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/4*a^2*x^2*arcsinh(a*x)^3*(a^2*x^2+1)-1/4*arcsinh(a*x)^3*(a^2*x^2+1
)-3/16*arcsinh(a*x)^2*a*x*(a^2*x^2+1)^(3/2)+15/32*arcsinh(a*x)^2*a*x*(a^2*x
^2+1)^(1/2)+5/32*arcsinh(a*x)^3+3/32*arcsinh(a*x)*a^2*x^2*(a^2*x^2+1)-3/128
*a*x*(a^2*x^2+1)^(3/2)+51/256*a*x*(a^2*x^2+1)^(1/2)+51/256*arcsinh(a*x)-3/8
*(a^2*x^2+1)*arcsinh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,x^4\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\int\frac{3\left(a^3x^6+\sqrt{a^2x^2+1}a^2x^5+ax^4\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\left(a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/4*(a^3*x^6 + sqrt(a^2*
x^2 + 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.14171, size = 327, normalized size = 2.01

$$\frac{8\left(8\,a^{4}x^{4}-3\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}-24\left(2\,a^{3}x^{3}-3\,ax\right)\sqrt{a^{2}x^{2}+1}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}+3\left(8\,a^{4}x^{4}-24\,a^{2}x^{2}-15\right)\log}{256\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 24*(2*a^3*x^3 - 3
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 3*(8*a^4*x^4 - 24*
a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^3*x^3 - 15*a*x)*sqrt(a^
2*x^2 + 1))/a^4
```

Sympy [A] time = 4.26919, size = 160, normalized size = 0.98

$$\begin{cases}\frac{x^{4}\operatorname{asinh}^{3}{(ax)}}{4}+\frac{3x^{4}\operatorname{asinh}{(ax)}}{32}-\frac{3x^{3}\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{2}{(ax)}}{16a}-\frac{3x^{3}\sqrt{a^{2}x^{2}+1}}{128a}-\frac{9x^{2}\operatorname{asinh}{(ax)}}{32a^{2}}+\frac{9x\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{2}{(ax)}}{32a^{3}}+\frac{45x\sqrt{a^{2}x^{2}+1}}{256a^{3}}-\frac{3\operatorname{asinh}^{3}{(a}}{32a^{4}} \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**3/4 + 3*x**4*asinh(a*x)/32 - 3*x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**2/(16*a) - 3*x**3*sqrt(a**2*x**2 + 1)/(128*a) - 9*x**2
*asinh(a*x)/(32*a**2) + 9*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(32*a**3) + 4
5*x*sqrt(a**2*x**2 + 1)/(256*a**3) - 3*asinh(a*x)**3/(32*a**4) - 45*asinh(a
*x)/(256*a**4), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\operatorname{arsinh}\left(ax\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^3, x)
```

3.24 $\int x^2 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=132

$$-\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^3}+\frac{14\sqrt{a^2x^2+1}}{9a^3}-\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a}+\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^3}-\frac{4x\sinh^{-1}(ax)}{3a^2}+\frac{1}{3}x^3\sinh^{-1}(ax$$

```
[Out] (14*Sqrt[1 + a^2*x^2])/(9*a^3) - (2*(1 + a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cSinh[a*x])/(3*a^2) + (2*x^3*ArcSinh[a*x])/9 + (2*Sqrt[1 + a^2*x^2]*ArcSinh
[a*x]^2)/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a) + (x^3*ArcS
inh[a*x]^3)/3
```

Rubi [A] time = 0.22147, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5661, 5758, 5717, 5653, 261, 266, 43}

$$-\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^3}+\frac{14\sqrt{a^2x^2+1}}{9a^3}-\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a}+\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^3}-\frac{4x\sinh^{-1}(ax)}{3a^2}+\frac{1}{3}x^3\sinh^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^3,x]
```


```
[Out] (14*Sqrt[1 + a^2*x^2])/(9*a^3) - (2*(1 + a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cSinh[a*x])/(3*a^2) + (2*x^3*ArcSinh[a*x])/9 + (2*Sqrt[1 + a^2*x^2]*ArcSinh
[a*x]^2)/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a) + (x^3*ArcS
inh[a*x]^3)/3
```


Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^3 \, dx = \frac{1}{3}x^3 \sinh^{-1}(ax)^3 - a \int \frac{x^3 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \sinh^{-1}(ax)^3 + \frac{2}{3}\int x^2 \sinh^{-1}(ax)\, dx + \frac{2\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\, dx}{3a}$$

$$= \frac{2}{9}x^3 \sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \sinh^{-1}(ax)^3 - \frac{4\int \mathrm{s}}{}$$

$$= -\frac{4x\sinh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \mathrm{s}$$

$$= \frac{4\sqrt{1+a^2x^2}}{3a^3} - \frac{4x\sinh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(}{3a}$$

$$= \frac{14\sqrt{1+a^2x^2}}{9a^3} - \frac{2\left(1+a^2x^2\right)^{3/2}}{27a^3} - \frac{4x\sinh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2}{}$$

Mathematica [A] time = 0.0542938, size = 93, normalized size = 0.7

$$\frac{-2\left(a^2x^2-20\right)\sqrt{a^2x^2+1}+9a^3x^3\sinh^{-1}(ax)^3-9\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2+6ax\left(a^2x^2-6\right)\sinh^{-1}(ax)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^3,x]
```

```
[Out] (-2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] + 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] -
 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 9*a^3*x^3*ArcSinh[a*x]
^3)/(27*a^3)
```

Maple [A] time = 0.028, size = 136, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{(\mathrm{Arcsinh}\,(ax))^3\, ax\left(a^2x^2+1\right)}{3} - \frac{(\mathrm{Arcsinh}\,(ax))^3\, ax}{3} - \frac{a^2x^2\,(\mathrm{Arcsinh}\,(ax))^2}{3}\sqrt{a^2x^2+1} + \frac{2\,(\mathrm{Arcsinh}\,(ax))^2}{3}\sqrt{a^2x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/3*arcsinh(a*x)^3*a*x*(a^2*x^2+1)-1/3*arcsinh(a*x)^3*a*x-1/3*a^2*x^
2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+2/9
*arcsinh(a*x)*a*x*(a^2*x^2+1)-14/9*a*x*arcsinh(a*x)-2/27*a^2*x^2*(a^2*x^2+1
)^(1/2)+40/27*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.14105, size = 157, normalized size = 1.19

$$\frac{1}{3}x^3\,\mathrm{arsinh}\,(ax)^3 - \frac{1}{3}a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\mathrm{arsinh}\,(ax)^2 - \frac{2}{27}a\left(\frac{\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}}{a^2} - \frac{3\left(a^2x^3-6x\right)\mathrm{ar}}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^3 - 1/3*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x)^2 - 2/27*a*((sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2
 + 1)/a^2)/a^2 - 3*(a^2*x^3 - 6*x)*arcsinh(a*x)/a^3)
```

Fricas [A] time = 2.11573, size = 281, normalized size = 2.13

$$\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 9\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 6\left(a^3x^3-6\,ax\right)\log\left(ax+\sqrt{a^2x^2+1}\right) - 2}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x
^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*x^3 - 6*a*x)*log(a*x + sqrt
(a^2*x^2 + 1)) - 2*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20))/a^3
```

Sympy [A] time = 2.23995, size = 128, normalized size = 0.97

$$\begin{cases} \frac{x^3\,\mathrm{asinh}^3\,(ax)}{3} + \frac{2x^3\,\mathrm{asinh}\,(ax)}{9} - \frac{x^2\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{3a} - \frac{2x^2\sqrt{a^2x^2+1}}{27a} - \frac{4x\,\mathrm{asinh}\,(ax)}{3a^2} + \frac{2\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{3a^3} + \frac{40\sqrt{a^2x^2+1}}{27a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**3/3 + 2*x**3*asinh(a*x)/9 - x**2*sqrt(a**2*x**2
 + 1)*asinh(a*x)**2/(3*a) - 2*x**2*sqrt(a**2*x**2 + 1)/(27*a) - 4*x*asinh(a
*x)/(3*a**2) + 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**3) + 40*sqrt(a**2*
x**2 + 1)/(27*a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.56615, size = 190, normalized size = 1.44

$$\frac{1}{3}\,x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\frac{1}{27}\,a\left(\frac{9\left(\left(a^2x^2+1\right)^{\frac{3}{2}}-3\,\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^4}-\frac{2\left(3\left(a^2x^3-6\,x\right)\log\left(ax+\sqrt{\right.\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^3 - 1/27*a*(9*((a^2*x^2 + 1)^(3/2) - 3
*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2/a^4 - 2*(3*(a^2*x^3 - 6*
x)*log(a*x + sqrt(a^2*x^2 + 1)) - ((a^2*x^2 + 1)^(3/2) - 21*sqrt(a^2*x^2 +
1))/a)/a^3)
```

3.25 $\int x \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=97

$$-\frac{3x\sqrt{a^2x^2+1}}{8a}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{3\sinh^{-1}(ax)}{8a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^3+\frac{3}{4}x^2\sinh^{-1}(ax)$$

```
[Out] (-3*x*Sqrt[1 + a^2*x^2])/(8*a) + (3*ArcSinh[a*x])/(8*a^2) + (3*x^2*ArcSinh[
a*x])/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*a) + ArcSinh[a*x]^3/(4*
a^2) + (x^2*ArcSinh[a*x]^3)/2
```

Rubi [A] time = 0.152615, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5661, 5758, 5675, 321, 215}

$$-\frac{3x\sqrt{a^2x^2+1}}{8a}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{3\sinh^{-1}(ax)}{8a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^3+\frac{3}{4}x^2\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*x*Sqrt[1 + a^2*x^2])/(8*a) + (3*ArcSinh[a*x])/(8*a^2) + (3*x^2*ArcSinh[
a*x])/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*a) + ArcSinh[a*x]^3/(4*
a^2) + (x^2*ArcSinh[a*x]^3)/2
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}(ax)^3\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax)^3-\frac{1}{2}(3a)\int\frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4a}+\frac{1}{2}x^2\sinh^{-1}(ax)^3+\frac{3}{2}\int x\sinh^{-1}(ax)\,dx+\frac{3\int\frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a}\\
&= \frac{3}{4}x^2\sinh^{-1}(ax)-\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^3-\frac{1}{4}(3a)\int\frac{x^2}{\sqrt{1+a^2x^2}}\\
&= -\frac{3x\sqrt{1+a^2x^2}}{8a}+\frac{3}{4}x^2\sinh^{-1}(ax)-\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^3+\\
&= -\frac{3x\sqrt{1+a^2x^2}}{8a}+\frac{3\sinh^{-1}(ax)}{8a^2}+\frac{3}{4}x^2\sinh^{-1}(ax)-\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{1}{2}x^2
\end{aligned}$$

Mathematica [A] time = 0.0435968, size = 80, normalized size = 0.82

$$\frac{-3ax\sqrt{a^2x^2+1}+\left(4a^2x^2+2\right)\sinh^{-1}(ax)^3-6ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2+\left(6a^2x^2+3\right)\sinh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x*Sqrt[1 + a^2*x^2] + (3 + 6*a^2*x^2)*ArcSinh[a*x] - 6*a*x*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^2 + (2 + 4*a^2*x^2)*ArcSinh[a*x]^3)/(8*a^2)
```

Maple [A] time = 0.025, size = 88, normalized size = 0.9

$$\frac{1}{a^2}\left(\frac{(\operatorname{Arcsinh}(ax))^3\left(a^2x^2+1\right)}{2}-\frac{3\,(\operatorname{Arcsinh}(ax))^2\,ax}{4}\sqrt{a^2x^2+1}-\frac{(\operatorname{Arcsinh}(ax))^3}{4}+\frac{\left(3\,a^2x^2+3\right)\operatorname{Arcsinh}(ax)}{4}-\frac{3\,a}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^2*(1/2*arcsinh(a*x)^3*(a^2*x^2+1)-3/4*arcsinh(a*x)^2*a*x*(a^2*x^2+1)^(1
/2)-1/4*arcsinh(a*x)^3+3/4*(a^2*x^2+1)*arcsinh(a*x)-3/8*a*x*(a^2*x^2+1)^(1/
2)-3/8*arcsinh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,x^2\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\int\frac{3\left(a^3x^4+\sqrt{a^2x^2+1}a^2x^3+ax^2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\left(a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/2*(a^3*x^4 + sqrt(a^2*
x^2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.08086, size = 261, normalized size = 2.69

$$-\frac{6\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+3\sqrt{a^2x^2+1}ax-3\left(2\,a^2x^2+1\right)\log\left(ax\right.}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*(2*a^2*x^2
 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] time = 1.11925, size = 92, normalized size = 0.95

$$\begin{cases} \frac{x^2 \operatorname{asinh}^3(ax)}{2} + \frac{3x^2 \operatorname{asinh}(ax)}{4} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a} - \frac{3x\sqrt{a^2x^2+1}}{8a} + \frac{\operatorname{asinh}^3(ax)}{4a^2} + \frac{3\operatorname{asinh}(ax)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**3/2 + 3*x**2*asinh(a*x)/4 - 3*x*sqrt(a**2*x**2
+ 1)*asinh(a*x)**2/(4*a) - 3*x*sqrt(a**2*x**2 + 1)/(8*a) + asinh(a*x)**3/(4
*a**2) + 3*asinh(a*x)/(8*a**2), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^3, x)
```

3.26 $\int \sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=58

$$-\frac{6\sqrt{a^2x^2+1}}{a}-\frac{3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6x\sinh^{-1}(ax)$$

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

Rubi [A] time = 0.0808533, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5653, 5717, 261}

$$-\frac{6\sqrt{a^2x^2+1}}{a}-\frac{3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6x\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(ax)^3\,dx &= x\sinh^{-1}(ax)^3-(3a)\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6\int \sinh^{-1}(ax)\,dx\\
&= 6x\sinh^{-1}(ax)-\frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3-(6a)\int \frac{x}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{6\sqrt{1+a^2x^2}}{a}+6x\sinh^{-1}(ax)-\frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3
\end{aligned}$$

Mathematica [A] time = 0.0160733, size = 58, normalized size = 1.

$$-\frac{6\sqrt{a^2x^2+1}}{a}-\frac{3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6x\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

Maple [A] time = 0.027, size = 55, normalized size = 1.

$$\frac{1}{a}\left((\mathrm{Arcsinh}\,(ax))^3\,ax-3\,(\mathrm{Arcsinh}\,(ax))^2\,\sqrt{a^2x^2+1}+6\,ax\mathrm{Arcsinh}\,(ax)-6\,\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3,x)
```

```
[Out] 1/a*(arcsinh(a*x)^3*a*x-3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+6*a*x*arcsinh(a*
x)-6*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.3084, size = 77, normalized size = 1.33

$$x \operatorname{arsinh}(ax)^3 - \frac{3\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^2}{a} + \frac{6\left(ax \operatorname{arsinh}(ax) - \sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^3 - 3*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a + 6*(a*x*arcsinh(a*
x) - sqrt(a^2*x^2 + 1))/a
```

Fricas [A] time = 2.0196, size = 205, normalized size = 3.53

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 6\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) - 3\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 + 6*a*x*log(a*x + sqrt(a^2*x^2 + 1)) -
3*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*x^2 + 1))/a
```

Sympy [A] time = 0.517625, size = 54, normalized size = 0.93

$$\begin{cases} x \operatorname{asinh}^3(ax) + 6x\operatorname{asinh}(ax) - \frac{3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{a} - \frac{6\sqrt{a^2x^2+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3,x)
```

```
[Out] Piecewise((x*asinh(a*x)**3 + 6*x*asinh(a*x) - 3*sqrt(a**2*x**2 + 1)*asinh(a
*x)**2/a - 6*sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

<u>Giac</u> [A] time = 1.43003, size = 132, normalized size = 2.28

$$x \log\left(ax + \sqrt{a^2x^2 + 1}\right)^3 - 3\,a\left(\frac{\sqrt{a^2x^2 + 1}\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{a^2} - \frac{2\left(x \log\left(ax + \sqrt{a^2x^2 + 1}\right) - \frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^
2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
)/a)
```

3.27 $\int \frac{\sinh^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=83

$$\frac{3}{2}\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}\sinh^{-1}(ax)\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)+\frac{3}{4}\text{PolyLog}\left(4,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{4}\sinh^{-}$$

```
[Out] -ArcSinh[a*x]^4/4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSinh
[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^(2
*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

Rubi [A] time = 0.112335, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}\sinh^{-1}(ax)\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)+\frac{3}{4}\text{PolyLog}\left(4,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{4}\sinh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x,x]
```


```
[Out] -ArcSinh[a*x]^4/4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSinh
[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^(2
*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```


Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x}\,dx = \text{Subst}\left(\int x^3\coth(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{1}{4}\sinh^{-1}(ax)^4 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^3}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) - 3\,\text{Subst}\left(\int x^2\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}(ax\right.$$
$$= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\,\text{Subst}\left(\int\right.$$
$$= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\sinh^{-1}(a$$
$$= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\sinh^{-1}(a$$
$$= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\sinh^{-1}(a$$

Mathematica [A] time = 0.0070033, size = 83, normalized size = 1.

$$\frac{3}{2}\sinh^{-1}(ax)^2\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\sinh^{-1}(ax)\text{PolyLog}\left(3, e^{2\sinh^{-1}(ax)}\right) + \frac{3}{4}\text{PolyLog}\left(4, e^{2\sinh^{-1}(ax)}\right) - \frac{1}{4}\sinh^{-}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/x,x]
```

```
[Out] -ArcSinh[a*x]^4/4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSinh
[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^(2
*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

Maple [A] time = 0.036, size = 204, normalized size = 2.5

$$-\frac{(\text{Arcsinh}\,(ax))^4}{4} + (\text{Arcsinh}\,(ax))^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 3\,(\text{Arcsinh}\,(ax))^2\,\text{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right) - 6\,\text{Ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x,x)
```

```
[Out] -1/4*arcsinh(a*x)^4+arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))+3*arcsinh(a*
x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x
```

```
^2+1)^(1/2))+6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a
^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsin
h(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x,x)
```

```
[Out] Integral(asinh(a*x)**3/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x, x)
```

3.28 $\int \frac{\sinh^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=84

$$-6a\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+6a\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+6a\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)-6a\text{PolyL}$$

```
[Out] -(ArcSinh[a*x]^3/x) - 6*a*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 6*a*ArcS
inh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] + 6*a*ArcSinh[a*x]*PolyLog[2, E^ArcSin
h[a*x]] + 6*a*PolyLog[3, -E^ArcSinh[a*x]] - 6*a*PolyLog[3, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.159138, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5661, 5760, 4182, 2531, 2282, 6589}

$$-6a\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+6a\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+6a\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)-6a\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^2,x]
```


```
[Out] -(ArcSinh[a*x]^3/x) - 6*a*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 6*a*ArcS
inh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] + 6*a*ArcSinh[a*x]*PolyLog[2, E^ArcSin
h[a*x]] + 6*a*PolyLog[3, -E^ArcSinh[a*x]] - 6*a*PolyLog[3, E^ArcSinh[a*x]]
```


Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^2}\,dx = -\frac{\sinh^{-1}(ax)^3}{x} + (3a)\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\sinh^{-1}(ax)^3}{x} + (3a)\,\mathrm{Subst}\left(\int x^2\mathbf{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (6a)\,\mathrm{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right.$$
$$= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 6a\sinh^{-1}(ax)\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 6a\sinh^{-1}(ax$$
$$= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 6a\sinh^{-1}(ax)\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 6a\sinh^{-1}(ax$$
$$= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 6a\sinh^{-1}(ax)\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 6a\sinh^{-1}(ax$$

Mathematica [A] time = 0.115774, size = 117, normalized size = 1.39

$$a\left(6\sinh^{-1}(ax)\mathbf{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) - 6\sinh^{-1}(ax)\mathbf{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right) + 6\mathbf{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right) - 6\mathbf{Poly}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^2,x]
```

```
[Out] a*(-(ArcSinh[a*x]^3/(a*x)) + 3*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] -
3*ArcSinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 6*ArcSinh[a*x]*PolyLog[2, -E^
(-ArcSinh[a*x])] - 6*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])] + 6*PolyLog
[3, -E^(-ArcSinh[a*x])] - 6*PolyLog[3, E^(-ArcSinh[a*x])])
```

Maple [A] time = 0.049, size = 162, normalized size = 1.9

$$-\frac{(\mathrm{Arcsinh}\,(ax))^3}{x} - 3\,a\,(\mathrm{Arcsinh}\,(ax))^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 6\,a\mathrm{Arcsinh}\,(ax)\,\mathrm{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right) + 6\,ap$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^2,x)
```

```
[Out] -arcsinh(a*x)^3/x-3*a*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))-6*a*arcsin
h(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/
```

```
2))+3*a*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog
(2,a*x+(a^2*x^2+1)^(1/2))-6*a*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{x}+\int\frac{3\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^3x^4+ax^2+\left(a^2x^3+x\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**2,x)
```

```
[Out] Integral(asinh(a*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^2, x)
```

3.29 $\int \frac{\sinh^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=93

$$\frac{3}{2}a^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2x}-\frac{3}{2}a^2\sinh^{-1}(ax)^2+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-\mathrm{sin}$$

```
[Out] (-3*a^2*ArcSinh[a*x]^2)/2 - (3*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x) -
ArcSinh[a*x]^3/(2*x^2) + 3*a^2*ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + (
3*a^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rubi [A] time = 0.168133, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5661, 5723, 5659, 3716, 2190, 2279, 2391}

$$\frac{3}{2}a^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2x}-\frac{3}{2}a^2\sinh^{-1}(ax)^2+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-\mathrm{sin}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^3,x]
```

```
[Out] (-3*a^2*ArcSinh[a*x]^2)/2 - (3*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x) -
ArcSinh[a*x]^3/(2*x^2) + 3*a^2*ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + (
3*a^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
```

```
NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^3}\,dx = -\frac{\sinh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{3a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{2x^2} + \left(3a^2\right)\int \frac{\sinh^{-1}(ax)}{x}\,dx$$
$$= -\frac{3a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{2x^2} + \left(3a^2\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{3}{2}a^2\sinh^{-1}(ax)^2 - \frac{3a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{2x^2} - \left(6a^2\right)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh$$
$$= -\frac{3}{2}a^2\sinh^{-1}(ax)^2 - \frac{3a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{2x^2} + 3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right.$$
$$= -\frac{3}{2}a^2\sinh^{-1}(ax)^2 - \frac{3a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{2x^2} + 3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right.$$
$$= -\frac{3}{2}a^2\sinh^{-1}(ax)^2 - \frac{3a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{2x^2} + 3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right.$$

Mathematica [A] time = 0.317221, size = 80, normalized size = 0.86

$$-\frac{\sinh^{-1}(ax)^3 - 3ax\left(\sinh^{-1}(ax)\left(\left(ax-\sqrt{a^2x^2+1}\right)\sinh^{-1}(ax) + 2ax\log\left(1-e^{-2\sinh^{-1}(ax)}\right)\right) - ax\text{PolyLog}\left(2, e^{-2\sinh^{-1}(\right.\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^3,x]
```

```
[Out] -(ArcSinh[a*x]^3 - 3*a*x*(ArcSinh[a*x]*((a*x - Sqrt[1 + a^2*x^2])*ArcSinh[a
*x] + 2*a*x*Log[1 - E^(-2*ArcSinh[a*x])]) - a*x*PolyLog[2, E^(-2*ArcSinh[a*
x])]))/(2*x^2)
```

Maple [A] time = 0.068, size = 149, normalized size = 1.6

$$-\frac{3\,a^2\left(\text{Arcsinh}\,(ax)\right)^2}{2} - \frac{3\,a\left(\text{Arcsinh}\,(ax)\right)^2}{2\,x}\sqrt{a^2x^2+1} - \frac{\left(\text{Arcsinh}\,(ax)\right)^3}{2\,x^2} + 3\,a^2\text{Arcsinh}\,(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^3,x)
```

```
[Out] -3/2*a^2*arcsinh(a*x)^2-3/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x-1/2*arcsin
h(a*x)^3/x^2+3*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+3*a^2*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a
^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{2\,x^2}+\int\frac{3\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\left(a^3x^5+ax^3+\left(a^2x^4+x^2\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a^2*x^2 + 1))^3/x^2 + integrate(3/2*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^5 + a*x^3 + (a^2
*x^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{arsinh}\left(ax\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}^3\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**3,x)
```

```
[Out] Integral(asinh(a*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^3, x)
```

3.30 $\int \frac{\sinh^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=151

$$a^3 \sinh^{-1}(ax)\text{PolyLog}\left(2, -e^{\sinh^{-1}(ax)}\right) - a^3 \sinh^{-1}(ax)\text{PolyLog}\left(2, e^{\sinh^{-1}(ax)}\right) - a^3\text{PolyLog}\left(3, -e^{\sinh^{-1}(ax)}\right) + a^3\text{PolyLo}$$

```
[Out] -((a^2*ArcSinh[a*x])/x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) - Ar
cSinh[a*x]^3/(3*x^3) + a^3*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^3*Arc
Tanh[Sqrt[1 + a^2*x^2]] + a^3*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^
3*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] - a^3*PolyLog[3, -E^ArcSinh[a*x]]
 + a^3*PolyLog[3, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.277181, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5661, 5747, 5760, 4182, 2531, 2282, 6589, 266, 63, 208}

$$a^3 \sinh^{-1}(ax)\text{PolyLog}\left(2, -e^{\sinh^{-1}(ax)}\right) - a^3 \sinh^{-1}(ax)\text{PolyLog}\left(2, e^{\sinh^{-1}(ax)}\right) - a^3\text{PolyLog}\left(3, -e^{\sinh^{-1}(ax)}\right) + a^3\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^4,x]
```


```
[Out] -((a^2*ArcSinh[a*x])/x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) - Ar
cSinh[a*x]^3/(3*x^3) + a^3*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^3*Arc
Tanh[Sqrt[1 + a^2*x^2]] + a^3*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^
3*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] - a^3*PolyLog[3, -E^ArcSinh[a*x]]
 + a^3*PolyLog[3, E^ArcSinh[a*x]]
```


Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x^4}\,dx &= -\frac{\sinh^{-1}(ax)^3}{3x^3} + a\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^2\int \frac{\sinh^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^3\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} - \frac{1}{2}a^3\,\mathrm{Subst}\left(\int x^2\mathbf{csch}(x)\,dx, x, \sinh^{-1}(\right. \\
&= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}\cdots \\
&= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^3\cdots \\
&= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - a^3\cdots \\
&= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - a^3\cdots
\end{aligned}
$$

Mathematica [A] time = 2.21881, size = 268, normalized size = 1.77

$$\frac{1}{48}a^3\left(-48\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+48\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-48\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^4,x]
```

```
[Out] (a^3*(-24*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] + 4*ArcSinh[a*x]^3*Coth[ArcSinh
[a*x]/2] - 6*ArcSinh[a*x]^2*Csch[ArcSinh[a*x]/2]^2 - a*x*ArcSinh[a*x]^3*Csc
h[ArcSinh[a*x]/2]^4 - 24*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 24*Arc
Sinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 48*Log[Tanh[ArcSinh[a*x]/2]] - 48*
ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] + 48*ArcSinh[a*x]*PolyLog[2, E^
(-ArcSinh[a*x])] - 48*PolyLog[3, -E^(-ArcSinh[a*x])] + 48*PolyLog[3, E^(-Ar
cSinh[a*x])] - 6*ArcSinh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 - (16*ArcSinh[a*x]^3
*Sinh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2] -
 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/48
```

Maple [A] time = 0.092, size = 228, normalized size = 1.5

$$-\frac{a\left(\text{Arcsinh}\left(ax\right)\right)^2}{2x^2}\sqrt{a^2x^2+1}-\frac{a^2\text{Arcsinh}\left(ax\right)}{x}-\frac{\left(\text{Arcsinh}\left(ax\right)\right)^3}{3x^3}+\frac{a^3\left(\text{Arcsinh}\left(ax\right)\right)^2}{2}\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^4,x)
```

```
[Out] -1/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x^2-a^2*arcsinh(a*x)/x-1/3*arcsinh(
a*x)^3/x^3+1/2*a^3*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+a^3*arcsinh(a
*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^3*polylog(3,-a*x-(a^2*x^2+1)^(1/2))
-1/2*a^3*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-a^3*arcsinh(a*x)*polylo
g(2,a*x+(a^2*x^2+1)^(1/2))+a^3*polylog(3,a*x+(a^2*x^2+1)^(1/2))-2*a^3*arcta
nh(a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3x^3}+\int\frac{\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^3x^6+ax^4+\left(a^2x^5+x^3\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/x^3 + integrate((a^3*x^2 + sqrt(a^2*x^2
 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^6 + a*x^4 + (a^2*x^5
 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}\left(ax\right)^{3}}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{3}\left(ax\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**4,x)
```

```
[Out] Integral(asinh(a*x)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(ax\right)^{3}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^4, x)
```

3.31 $\int \frac{\sinh^{-1}(ax)^3}{x^5} dx$

Optimal. Leaf size=159

$$-\frac{1}{2}a^4\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{a^3\sqrt{a^2x^2+1}}{4x}+\frac{a^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{2x}-\frac{a^2\sinh^{-1}(ax)}{4x^2}-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{4x^3}+$$

```
[Out] -(a^3*Sqrt[1 + a^2*x^2])/(4*x) - (a^2*ArcSinh[a*x])/(4*x^2) + (a^4*ArcSinh[
a*x]^2)/2 - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^2)/(2*x) - ArcSinh[a*x]^3/(4*x^4) - a^4*ArcSinh[a*x]*Lo
g[1 - E^(2*ArcSinh[a*x])] - (a^4*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rubi [A] time = 0.285884, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {5661, 5747, 5723, 5659, 3716, 2190, 2279, 2391, 264}

$$-\frac{1}{2}a^4\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{a^3\sqrt{a^2x^2+1}}{4x}+\frac{a^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{2x}-\frac{a^2\sinh^{-1}(ax)}{4x^2}-\frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{4x^3}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^5,x]
```

```
[Out] -(a^3*Sqrt[1 + a^2*x^2])/(4*x) - (a^2*ArcSinh[a*x])/(4*x^2) + (a^4*ArcSinh[
a*x]^2)/2 - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^2)/(2*x) - ArcSinh[a*x]^3/(4*x^4) - a^4*ArcSinh[a*x]*Lo
g[1 - E^(2*ArcSinh[a*x])] - (a^4*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^5}\,dx = -\frac{\sinh^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^4\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} - \frac{\sinh^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)}{x^3}\,dx - \frac{1}{2}a^3\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a^2\sinh^{-1}(ax)}{4x^2} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^3\int \frac{}{x^2\sqrt{}}$$

$$= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)}{4x^4}$$

$$= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}}{2x}$$

$$= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}}{2x}$$

$$= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}}{2x}$$

$$= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}}{2x}$$

Mathematica [A] time = 0.606866, size = 107, normalized size = 0.67

$$\frac{1}{4}\left(a^4\left(2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(ax)}\right) - \frac{\sqrt{a^2x^2+1}\left(\left(\frac{1}{a^2x^2}-2\right)\sinh^{-1}(ax)^2+1\right)}{ax} - \sinh^{-1}(ax)\left(\frac{1}{a^2x^2} + 2\sinh^{-1}(ax) + 4\log\left(\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^5,x]
```

```
[Out] (-(ArcSinh[a*x]^3/x^4) + a^4*(-((Sqrt[1 + a^2*x^2]*(1 + (-2 + 1/(a^2*x^2))*
ArcSinh[a*x]^2))/(a*x)) - ArcSinh[a*x]*(1/(a^2*x^2) + 2*ArcSinh[a*x] + 4*Lo
g[1 - E^(-2*ArcSinh[a*x])]) + 2*PolyLog[2, E^(-2*ArcSinh[a*x])]))/4
```

Maple [A] time = 0.106, size = 210, normalized size = 1.3

$$\frac{a^4 (\mathrm{Arcsinh}(ax))^2}{2} + \frac{a^3 (\mathrm{Arcsinh}(ax))^2}{2x}\sqrt{a^2x^2+1} + \frac{a^4}{4} - \frac{a^3}{4x}\sqrt{a^2x^2+1} - \frac{a (\mathrm{Arcsinh}(ax))^2}{4x^3}\sqrt{a^2x^2+1} - \frac{a^2\mathrm{Arcsinh}(}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^5,x)
```

```
[Out] 1/2*a^4*arcsinh(a*x)^2+1/2*a^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x+1/4*a^4-1
/4*a^3*(a^2*x^2+1)^(1/2)/x-1/4*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x^3-1/4*a
^2*arcsinh(a*x)/x^2-1/4*arcsinh(a*x)^3/x^4-a^4*arcsinh(a*x)*ln(1+a*x+(a^2*x
^2+1)^(1/2))-a^4*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^4*arcsinh(a*x)*ln(1-a*
x-(a^2*x^2+1)^(1/2))-a^4*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{4x^4} + \int \frac{3\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\left(a^3x^7+ax^5+\left(a^2x^6+x^4\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log(a*x + sqrt(a^2*x^2 + 1))^3/x^4 + integrate(3/4*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^7 + a*x^5 + (a^2
*x^6 + x^4)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}(ax)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**5,x)
```

```
[Out] Integral(asinh(a*x)**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^5, x)
```

3.32 $\int x^5 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=276

$$-\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}-\frac{x^5\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{9a}-\frac{x^5\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{54a}-\frac{5x^4\sinh^{-1}(ax)^2}{48a^2}+\frac{5x^3\sqrt{a^2x^2+1}\sinh^{-1}(a}{36a^3}$$

```
[Out] (245*x^2)/(1152*a^4) - (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x])/(576*a^5) + (65*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(864
*a^3) - (x^5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(54*a) + (245*ArcSinh[a*x]^2)/
(1152*a^6) + (5*x^2*ArcSinh[a*x]^2)/(16*a^4) - (5*x^4*ArcSinh[a*x]^2)/(48*a
^2) + (x^6*ArcSinh[a*x]^2)/18 - (5*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(24*
a^5) + (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^3)/(9*a) + (5*ArcSinh[a*x]^4)/(96*a^6) + (x^6*ArcSinh[a
*x]^4)/6
```

Rubi [A] time = 0.855782, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5661, 5758, 5675, 30}

$$-\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}-\frac{x^5\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{9a}-\frac{x^5\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{54a}-\frac{5x^4\sinh^{-1}(ax)^2}{48a^2}+\frac{5x^3\sqrt{a^2x^2+1}\sinh^{-1}(a}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcSinh[a*x]^4,x]
```

```
[Out] (245*x^2)/(1152*a^4) - (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x])/(576*a^5) + (65*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(864
*a^3) - (x^5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(54*a) + (245*ArcSinh[a*x]^2)/
(1152*a^6) + (5*x^2*ArcSinh[a*x]^2)/(16*a^4) - (5*x^4*ArcSinh[a*x]^2)/(48*a
^2) + (x^6*ArcSinh[a*x]^2)/18 - (5*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(24*
a^5) + (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^3)/(9*a) + (5*ArcSinh[a*x]^4)/(96*a^6) + (x^6*ArcSinh[a
*x]^4)/6
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^5 \sinh^{-1}(ax)^4\,dx = \frac{1}{6}x^6\sinh^{-1}(ax)^4 - \frac{1}{3}(2a)\int \frac{x^6\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6\sinh^{-1}(ax)^4 + \frac{1}{3}\int x^5\sinh^{-1}(ax)^2\,dx + \frac{5\int \frac{x^4\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{9a}$$

$$= \frac{1}{18}x^6\sinh^{-1}(ax)^2 + \frac{5x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{36a^3} - \frac{x^5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6\sinh^{-1}(ax)^4 -$$

$$= -\frac{x^5\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{54a} - \frac{5x^4\sinh^{-1}(ax)^2}{48a^2} + \frac{1}{18}x^6\sinh^{-1}(ax)^2 - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{24a^5} +$$

$$= \frac{x^6}{324} + \frac{65x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{864a^3} - \frac{x^5\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{54a} + \frac{5x^2\sinh^{-1}(ax)^2}{16a^4} - \frac{5x^4\sinh^{-1}(a}{48a^2}$$

$$= -\frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{576a^5} + \frac{65x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{864a^3} - \frac{x^5\sqrt{1+a^2x^2}\sinh}{54a}$$

$$= \frac{245x^2}{1152a^4} - \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{576a^5} + \frac{65x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{864a^3} - \frac{x^5\sqrt{1+a}}{}$$

Mathematica [A] time = 0.0902957, size = 165, normalized size = 0.6

$$\frac{a^2x^2\left(32a^4x^4-195a^2x^2+2205\right)+108\left(16a^6x^6+5\right)\sinh^{-1}(ax)^4-144ax\sqrt{a^2x^2+1}\left(8a^4x^4-10a^2x^2+15\right)\sinh^{-1}(ax)^3}{10368a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcSinh[a*x]^4,x]
```

```
[Out] (a^2*x^2*(2205 - 195*a^2*x^2 + 32*a^4*x^4) - 6*a*x*Sqrt[1 + a^2*x^2]*(735 -
 130*a^2*x^2 + 32*a^4*x^4)*ArcSinh[a*x] + 9*(245 + 360*a^2*x^2 - 120*a^4*x^
4 + 64*a^6*x^6)*ArcSinh[a*x]^2 - 144*a*x*Sqrt[1 + a^2*x^2]*(15 - 10*a^2*x^2
 + 8*a^4*x^4)*ArcSinh[a*x]^3 + 108*(5 + 16*a^6*x^6)*ArcSinh[a*x]^4)/(10368*
a^6)
```

Maple [A] time = 0.16, size = 319, normalized size = 1.2

$$\frac{1}{a^6}\left(\frac{a^4x^4(\mathrm{Arcsinh}(ax))^4\left(a^2x^2+1\right)}{6} - \frac{(\mathrm{Arcsinh}(ax))^4a^2x^2\left(a^2x^2+1\right)}{6} + \frac{(\mathrm{Arcsinh}(ax))^4\left(a^2x^2+1\right)}{6} - \frac{(\mathrm{Arcsinh}(ax))}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^6*(1/6*a^4*x^4*arcsinh(a*x)^4*(a^2*x^2+1)-1/6*arcsinh(a*x)^4*a^2*x^2*(a
^2*x^2+1)+1/6*arcsinh(a*x)^4*(a^2*x^2+1)-1/9*arcsinh(a*x)^3*a^3*x^3*(a^2*x^
2+1)^(3/2)+1/4*arcsinh(a*x)^3*a*x*(a^2*x^2+1)^(3/2)-11/24*arcsinh(a*x)^3*a*
x*(a^2*x^2+1)^(1/2)-11/96*arcsinh(a*x)^4+1/18*arcsinh(a*x)^2*a^2*x^2*(a^2*x
^2+1)^2-31/144*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)+17/36*arcsinh(a*x)^2*(a^2
*x^2+1)-1/54*arcsinh(a*x)*a*x*(a^2*x^2+1)^(5/2)+97/864*arcsinh(a*x)*a*x*(a^
2*x^2+1)^(3/2)+1/324*a^2*x^2*(a^2*x^2+1)^2-259/10368*a^2*x^2*(a^2*x^2+1)+19
/81*a^2*x^2+19/81-299/576*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-299/1152*arcsi
nh(a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,x^6\log\left(ax+\sqrt{a^2x^2+1}\right)^4-\int\frac{2\left(a^3x^8+\sqrt{a^2x^2+1}a^2x^7+ax^6\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3\left(a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2/3*(a^3*x^8 + sqrt(a^2*
x^2 + 1)*a^2*x^7 + a*x^6)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.14026, size = 497, normalized size = 1.8

$$\frac{32\,a^6x^6-195\,a^4x^4+108\left(16\,a^6x^6+5\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^4-144\left(8\,a^5x^5-10\,a^3x^3+15\,ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/10368*(32*a^6*x^6 - 195*a^4*x^4 + 108*(16*a^6*x^6 + 5)*log(a*x + sqrt(a^2
*x^2 + 1))^4 - 144*(8*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*sqrt(a^2*x^2 + 1)*log(
a*x + sqrt(a^2*x^2 + 1))^3 + 2205*a^2*x^2 + 9*(64*a^6*x^6 - 120*a^4*x^4 + 3
60*a^2*x^2 + 245)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(32*a^5*x^5 - 130*a^3*
x^3 + 735*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^6
```

Sympy [A] time = 21.5359, size = 269, normalized size = 0.97

$$\begin{cases} \frac{x^6 \operatorname{asinh}^4(ax)}{6} + \frac{x^6 \operatorname{asinh}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{9a} - \frac{x^5\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{54a} - \frac{5x^4\operatorname{asinh}^2(ax)}{48a^2} - \frac{65x^4}{3456a^2} + \frac{5x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{36a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**6*asinh(a*x)**4/6 + x**6*asinh(a*x)**2/18 + x**6/324 - x**5*s
qrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - x**5*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(54*a) - 5*x**4*asinh(a*x)**2/(48*a**2) - 65*x**4/(3456*a**2) + 5*x**3*sq
rt(a**2*x**2 + 1)*asinh(a*x)**3/(36*a**3) + 65*x**3*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(864*a**3) + 5*x**2*asinh(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4)
- 5*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(24*a**5) - 245*x*sqrt(a**2*x**2 +
1)*asinh(a*x)/(576*a**5) + 5*asinh(a*x)**4/(96*a**6) + 245*asinh(a*x)**2/(1
152*a**6), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arsinh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^5*arcsinh(a*x)^4, x)
```

3.33 $\int x^4 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=244

$$-\frac{1088x^3}{16875a^2}-\frac{4x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{25a}-\frac{24x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{625a}-\frac{16x^3\sinh^{-1}(ax)^2}{75a^2}+\frac{16x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{75a^3}+$$

```
[Out] (16576*x)/(5625*a^4) - (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x])/(5625*a^3) - (24*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(625*a) + (32*
x*ArcSinh[a*x]^2)/(25*a^4) - (16*x^3*ArcSinh[a*x]^2)/(75*a^2) + (12*x^5*Arc
Sinh[a*x]^2)/125 - (32*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^5) + (16*x^2
*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 + a^2*x^2]*ArcS
inh[a*x]^3)/(25*a) + (x^5*ArcSinh[a*x]^4)/5
```

Rubi [A] time = 0.657166, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5661, 5758, 5717, 5653, 8, 30}

$$-\frac{1088x^3}{16875a^2}-\frac{4x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{25a}-\frac{24x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{625a}-\frac{16x^3\sinh^{-1}(ax)^2}{75a^2}+\frac{16x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{75a^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^4,x]
```

```
[Out] (16576*x)/(5625*a^4) - (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x])/(5625*a^3) - (24*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(625*a) + (32*
x*ArcSinh[a*x]^2)/(25*a^4) - (16*x^3*ArcSinh[a*x]^2)/(75*a^2) + (12*x^5*Arc
Sinh[a*x]^2)/125 - (32*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^5) + (16*x^2
*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 + a^2*x^2]*ArcS
inh[a*x]^3)/(25*a) + (x^5*ArcSinh[a*x]^4)/5
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^4\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^4 - \frac{1}{5}(4a)\int \frac{x^5\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{4x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^4 + \frac{12}{25}\int x^4\sinh^{-1}(ax)^2\,dx + \frac{16\int \frac{x^3\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{25a}$$

$$= \frac{12}{125}x^5\sinh^{-1}(ax)^2 + \frac{16x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{75a^3} - \frac{4x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^4$$

$$= -\frac{24x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{625a} - \frac{16x^3\sinh^{-1}(ax)^2}{75a^2} + \frac{12}{125}x^5\sinh^{-1}(ax)^2 - \frac{32\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{75a^5}$$

$$= \frac{24x^5}{3125} + \frac{1088x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{625a} + \frac{32x\sinh^{-1}(ax)^2}{25a^4} - \frac{16x^3\sinh}{75}$$

$$= -\frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2}}{62}$$

$$= \frac{16576x}{5625a^4} - \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{5625a^3} - \frac{24x^4}{}$$

Mathematica [A] time = 0.081171, size = 148, normalized size = 0.61

$$\frac{8ax\left(81a^4x^4 - 680a^2x^2 + 31080\right) + 16875a^5x^5\sinh^{-1}(ax)^4 + 900ax\left(9a^4x^4 - 20a^2x^2 + 120\right)\sinh^{-1}(ax)^2 - 4500\sqrt{a^2x^2+1}}{84375a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^4,x]
```

```
[Out] (8*a*x*(31080 - 680*a^2*x^2 + 81*a^4*x^4) - 120*Sqrt[1 + a^2*x^2]*(2072 - 1
36*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] + 900*a*x*(120 - 20*a^2*x^2 + 9*a^4*x
^4)*ArcSinh[a*x]^2 - 4500*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4)*Arc
Sinh[a*x]^3 + 16875*a^5*x^5*ArcSinh[a*x]^4)/(84375*a^5)
```

Maple [A] time = 0.04, size = 272, normalized size = 1.1

$$\frac{1}{a^5}\left(\frac{a^3x^3\left(\mathrm{Arcsinh}\,(ax)\right)^4\left(a^2x^2+1\right)}{5} - \frac{\left(\mathrm{Arcsinh}\,(ax)\right)^4 ax\left(a^2x^2+1\right)}{5} + \frac{\left(\mathrm{Arcsinh}\,(ax)\right)^4 ax}{5} - \frac{4\left(\mathrm{Arcsinh}\,(ax)\right)^3 a^2x^2}{25}\left(a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^5*(1/5*a^3*x^3*arcsinh(a*x)^4*(a^2*x^2+1)-1/5*arcsinh(a*x)^4*a*x*(a^2*x
^2+1)+1/5*arcsinh(a*x)^4*a*x-4/25*arcsinh(a*x)^3*a^2*x^2*(a^2*x^2+1)^(3/2)+
28/75*a^2*x^2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-32/75*arcsinh(a*x)^3*(a^2*x^
2+1)^(1/2)+12/125*arcsinh(a*x)^2*a*x*(a^2*x^2+1)^2+596/375*arcsinh(a*x)^2*a
*x-152/375*arcsinh(a*x)^2*a*x*(a^2*x^2+1)-24/625*arcsinh(a*x)*a^2*x^2*(a^2*
x^2+1)^(3/2)+1304/5625*arcsinh(a*x)*a^2*x^2*(a^2*x^2+1)^(1/2)-16576/5625*ar
csinh(a*x)*(a^2*x^2+1)^(1/2)+24/3125*a*x*(a^2*x^2+1)^2+254728/84375*a*x-673
6/84375*a*x*(a^2*x^2+1))
```

Maxima [A] time = 1.23611, size = 271, normalized size = 1.11

$$\frac{1}{5}x^5 \operatorname{arsinh}(ax)^4 - \frac{4}{75}\left(\frac{3\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{a^2x^2+1}}{a^6}\right)a \operatorname{arsinh}(ax)^3 - \frac{4}{84375}\left(2a\left(\frac{15\left(27\sqrt{a^2x^2+1}a\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^4 - 4/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^3 - 4/84375*(2*a*(1
5*(27*sqrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(a^2
*x^2 + 1)/a^2)*arcsinh(a*x)/a^5 - (81*a^4*x^5 - 680*a^2*x^3 + 31080*x)/a^6)
 - 225*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)^2/a^5)*a
```

Fricas [A] time = 2.14352, size = 466, normalized size = 1.91

$$\frac{16875\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 648\,a^5x^5 - 5440\,a^3x^3 - 4500\left(3\,a^4x^4 - 4\,a^2x^2 + 8\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/84375*(16875*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^4 + 648*a^5*x^5 - 5440*
a^3*x^3 - 4500*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
```

```
(a^2*x^2 + 1))^3 + 900*(9*a^5*x^5 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^
2*x^2 + 1))^2 - 120*(27*a^4*x^4 - 136*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1)*log
(a*x + sqrt(a^2*x^2 + 1)) + 248640*a*x)/a^5
```

Sympy [A] time = 12.4645, size = 241, normalized size = 0.99

$$\begin{cases} \frac{x^5 \operatorname{asinh}^4(ax)}{5} + \frac{12x^5 \operatorname{asinh}^2(ax)}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{25a} - \frac{24x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{625a} - \frac{16x^3\operatorname{asinh}^2(ax)}{75a^2} - \frac{1088x^3}{16875a^2} + \frac{16x^2\sqrt{a^2x^2+1}\operatorname{asin}}{75a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**4/5 + 12*x**5*asinh(a*x)**2/125 + 24*x**5/3125
- 4*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(25*a) - 24*x**4*sqrt(a**2*x**2
+ 1)*asinh(a*x)/(625*a) - 16*x**3*asinh(a*x)**2/(75*a**2) - 1088*x**3/(1687
5*a**2) + 16*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**3) + 1088*x**2*s
qrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**3) + 32*x*asinh(a*x)**2/(25*a**4) +
16576*x/(5625*a**4) - 32*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**5) - 1657
6*sqrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**5), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.81725, size = 293, normalized size = 1.2

$$\frac{1}{5}x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - \frac{4}{84375}a\left(\frac{1125\left(3\left(a^2x^2+1\right)^{\frac{5}{2}} - 10\left(a^2x^2+1\right)^{\frac{3}{2}} + 15\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^6} - \frac{16}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^4 - 4/84375*a*(1125*(3*(a^2*x^2 + 1)^(
5/2) - 10*(a^2*x^2 + 1)^(3/2) + 15*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^
2 + 1))^3/a^6 - (162*a^4*x^5 - 1360*a^2*x^3 + 225*(9*a^4*x^5 - 20*a^2*x^3 +
 120*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 62160*x - 30*(27*(a^2*x^2 + 1)^(5/
2) - 190*(a^2*x^2 + 1)^(3/2) + 2235*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x
^2 + 1))/a)/a^5)
```

3.34 $\int x^3 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=194

$$-\frac{45x^2}{128a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{4a}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{32a}-\frac{9x^2\sinh^{-1}(ax)^2}{16a^2}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{8a^3}+\frac{45x\sqrt{}}{}$$

```
[Out] (-45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(
64*a^3) - (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a) - (45*ArcSinh[a*x]^
2)/(128*a^4) - (9*x^2*ArcSinh[a*x]^2)/(16*a^2) + (3*x^4*ArcSinh[a*x]^2)/16
+ (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^3)/(4*a) - (3*ArcSinh[a*x]^4)/(32*a^4) + (x^4*ArcSinh[a*x]^4)/4
```

Rubi [A] time = 0.504404, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5661, 5758, 5675, 30}

$$-\frac{45x^2}{128a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{4a}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{32a}-\frac{9x^2\sinh^{-1}(ax)^2}{16a^2}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{8a^3}+\frac{45x\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^4,x]
```

```
[Out] (-45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(
64*a^3) - (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a) - (45*ArcSinh[a*x]^
2)/(128*a^4) - (9*x^2*ArcSinh[a*x]^2)/(16*a^2) + (3*x^4*ArcSinh[a*x]^2)/16
+ (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^3)/(4*a) - (3*ArcSinh[a*x]^4)/(32*a^4) + (x^4*ArcSinh[a*x]^4)/4
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\sinh^{-1}(ax)^4\,dx &= \frac{1}{4}x^4\sinh^{-1}(ax)^4 - a\int\frac{x^4\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\sinh^{-1}(ax)^4 + \frac{3}{4}\int x^3\sinh^{-1}(ax)^2\,dx + \frac{3\int\frac{x^2\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{4a}\\
&= \frac{3}{16}x^4\sinh^{-1}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{8a^3} - \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\sinh^{-1}(ax)^4 - \frac{3}{}\\
&= -\frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{32a} - \frac{9x^2\sinh^{-1}(ax)^2}{16a^2} + \frac{3}{16}x^4\sinh^{-1}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{8a^3} -\\
&= \frac{3x^4}{128} + \frac{45x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{32a} - \frac{9x^2\sinh^{-1}(ax)^2}{16a^2} + \frac{3}{16}x^4\sinh^{-1}(a\\
&= -\frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{32a} - \frac{45\sinh^{-1}(ax)^2}{128a^4} - \frac{9x^2 \mathrm{s}}{}\end{aligned}$$

Mathematica [A] time = 0.070216, size = 133, normalized size = 0.69

$$\frac{3a^2x^2\left(a^2x^2-15\right)+4\left(8a^4x^4-3\right)\sinh^{-1}(ax)^4-16ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^3+3\left(8a^4x^4-24a^2x^2-15\right)\sinh^{-}}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2*(-15 + a^2*x^2) - 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcS
inh[a*x] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4)*ArcSinh[a*x]^2 - 16*a*x*Sqrt[1
+ a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcSinh[a*x
]^4)/(128*a^4)
```

Maple [A] time = 0.036, size = 208, normalized size = 1.1

$$\frac{1}{a^4}\left(\frac{(\operatorname{Arcsinh}(ax))^4 a^2x^2\left(a^2x^2+1\right)}{4}-\frac{(\operatorname{Arcsinh}(ax))^4\left(a^2x^2+1\right)}{4}-\frac{(\operatorname{Arcsinh}(ax))^3 ax}{4}\left(a^2x^2+1\right)^{\frac{3}{2}}+\frac{5\,(\operatorname{Arcsinh}(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^4*(1/4*arcsinh(a*x)^4*a^2*x^2*(a^2*x^2+1)-1/4*arcsinh(a*x)^4*(a^2*x^2+1
)-1/4*arcsinh(a*x)^3*a*x*(a^2*x^2+1)^(3/2)+5/8*arcsinh(a*x)^3*a*x*(a^2*x^2+
1)^(1/2)+5/32*arcsinh(a*x)^4+3/16*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)-3/32*a
rcsinh(a*x)*a*x*(a^2*x^2+1)^(3/2)+51/64*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+
51/128*arcsinh(a*x)^2+3/128*a^2*x^2*(a^2*x^2+1)-3/4*arcsinh(a*x)^2*(a^2*x^2
+1)-3/8*a^2*x^2-3/8)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,x^4\log\left(ax+\sqrt{a^2x^2+1}\right)^4-\int\frac{\left(a^3x^6+\sqrt{a^2x^2+1}a^2x^5+ax^4\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate((a^3*x^6 + sqrt(a^2*x^2
+ 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^2*
x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.10842, size = 402, normalized size = 2.07

$$\frac{3\,a^4x^4+4\left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^4-16\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3-45\,a^2x^2+3\left(8\,a^4x^4\right.}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 16*(2
*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 45*a^2
*x^2 + 3*(8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(
2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] time = 7.5164, size = 190, normalized size = 0.98

$$\begin{cases}\frac{x^4\operatorname{asinh}^4(ax)}{4}+\frac{3x^4\operatorname{asinh}^2(ax)}{16}+\frac{3x^4}{128}-\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a}-\frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a}-\frac{9x^2\operatorname{asinh}^2(ax)}{16a^2}-\frac{45x^2}{128a^2}+\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{8a^3}+ \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**4/4 + 3*x**4*asinh(a*x)**2/16 + 3*x**4/128 - x*
*3*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(4*a) - 3*x**3*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(32*a) - 9*x**2*asinh(a*x)**2/(16*a**2) - 45*x**2/(128*a**2) + 3*x*
sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(8*a**3) + 45*x*sqrt(a**2*x**2 + 1)*asinh
(a*x)/(64*a**3) - 3*asinh(a*x)**4/(32*a**4) - 45*asinh(a*x)**2/(128*a**4),
Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{arsinh}(ax)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^4, x)
```

3.35 $\int x^2 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=162

$$-\frac{4x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{9a}+\frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{9a^3}-\frac{8x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{27a}+\frac{160\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{27a^3}-\frac{160x}{27a^2}$$

```
[Out] (-160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a
^3) - (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) - (8*x*ArcSinh[a*x]^2)/
(3*a^2) + (4*x^3*ArcSinh[a*x]^2)/9 + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(
9*a^3) - (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(9*a) + (x^3*ArcSinh[a*x]
^4)/3
```

Rubi [A] time = 0.363075, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5661, 5758, 5717, 5653, 8, 30}

$$-\frac{4x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{9a}+\frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{9a^3}-\frac{8x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{27a}+\frac{160\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{27a^3}-\frac{160x}{27a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^4,x]
```

```
[Out] (-160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a
^3) - (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) - (8*x*ArcSinh[a*x]^2)/
(3*a^2) + (4*x^3*ArcSinh[a*x]^2)/9 + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(
9*a^3) - (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(9*a) + (x^3*ArcSinh[a*x]
^4)/3
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^4\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^4 - \frac{1}{3}(4a)\int \frac{x^3\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{4x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^4 + \frac{4}{3}\int x^2\sinh^{-1}(ax)^2\,dx + \frac{8\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{9a}$$

$$= \frac{4}{9}x^3\sinh^{-1}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{9a^3} - \frac{4x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^4 - \frac{8\int$$

$$= -\frac{8x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{27a} - \frac{8x\sinh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3\sinh^{-1}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{9a^3} - \frac{4x^2$$

$$= \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{27a} - \frac{8x\sinh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3\sinh^{-1}(ax)^2$$

$$= -\frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{27a} - \frac{8x\sinh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3\,\mathrm{si}$$

Mathematica [A] time = 0.0696076, size = 112, normalized size = 0.69

$$\frac{8ax\left(a^2x^2-60\right)+27a^3x^3\sinh^{-1}(ax)^4-36\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3+36ax\left(a^2x^2-6\right)\sinh^{-1}(ax)^2-24\left(a^2x^2-\right.}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^4,x]
```

```
[Out] (8*a*x*(-60 + a^2*x^2) - 24*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
+ 36*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 - 36*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2
]*ArcSinh[a*x]^3 + 27*a^3*x^3*ArcSinh[a*x]^4)/(81*a^3)
```

Maple [A] time = 0.03, size = 165, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{(\mathrm{Arcsinh}\,(ax))^4\,ax\left(a^2x^2+1\right)}{3} - \frac{(\mathrm{Arcsinh}\,(ax))^4\,ax}{3} - \frac{4\,a^2x^2\,(\mathrm{Arcsinh}\,(ax))^3}{9}\sqrt{a^2x^2+1} + \frac{8\,(\mathrm{Arcsinh}\,(ax))^3}{9}\sqrt{a^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^3*(1/3*arcsinh(a*x)^4*a*x*(a^2*x^2+1)-1/3*arcsinh(a*x)^4*a*x-4/9*a^2*x^
2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+8/9*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+4/9
*arcsinh(a*x)^2*a*x*(a^2*x^2+1)-28/9*arcsinh(a*x)^2*a*x-8/27*arcsinh(a*x)*a
^2*x^2*(a^2*x^2+1)^(1/2)+160/27*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+8/81*a*x*(a^
2*x^2+1)-488/81*a*x)
```

Maxima [A] time = 1.23503, size = 193, normalized size = 1.19

$$\frac{1}{3}x^3\operatorname{arsinh}(ax)^4 - \frac{4}{9}a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^3 - \frac{4}{81}\left(2a\left(\frac{3\left(\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsinh}(ax)}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^4 - 4/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x)^3 - 4/81*(2*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^
2*x^2 + 1)/a^2)*arcsinh(a*x)/a^3 - (a^2*x^3 - 60*x)/a^4) - 9*(a^2*x^3 - 6*x
)*arcsinh(a*x)^2/a^3)*a
```

Fricas [A] time = 2.14329, size = 358, normalized size = 2.21

$$\frac{27a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 8a^3x^3 - 36\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 36\left(a^3x^3-6ax\right)\log\left(ax+\sqrt{\right.}\right.}{81a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^4 + 8*a^3*x^3 - 36*sqrt(a^2*x
^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 36*(a^3*x^3 - 6*a*x)
*log(a*x + sqrt(a^2*x^2 + 1))^2 - 24*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20)*log(a
*x + sqrt(a^2*x^2 + 1)) - 480*a*x)/a^3
```

Sympy [A] time = 4.24365, size = 158, normalized size = 0.98

$$\begin{cases} \frac{x^3 \operatorname{asinh}^4(ax)}{3} + \frac{4x^3 \operatorname{asinh}^2(ax)}{9} + \frac{8x^3}{81} - \frac{4x^2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{9a} - \frac{8x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{27a} - \frac{8x\operatorname{asinh}^2(ax)}{3a^2} - \frac{160x}{27a^2} + \frac{8\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{9a^3} + 1 \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**4/3 + 4*x**3*asinh(a*x)**2/9 + 8*x**3/81 - 4*x*
*2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - 8*x**2*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(27*a) - 8*x*asinh(a*x)**2/(3*a**2) - 160*x/(27*a**2) + 8*sqrt(a**2
*x**2 + 1)*asinh(a*x)**3/(9*a**3) + 160*sqrt(a**2*x**2 + 1)*asinh(a*x)/(27*
a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.80913, size = 230, normalized size = 1.42

$$\frac{1}{3}\,x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - \frac{4}{81}\,a\left(\frac{9\left(\left(a^2x^2+1\right)^{\frac{3}{2}} - 3\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^4} - \frac{2\,a^2x^3 + 9\left(a^2x^3 - 6\,x\right)\log\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^4 - 4/81*a*(9*((a^2*x^2 + 1)^(3/2) - 3
*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3/a^4 - (2*a^2*x^3 + 9*(a^
2*x^3 - 6*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 120*x - 6*((a^2*x^2 + 1)^(3/2
) - 21*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))/a)/a^3)
```

3.36 $\int x \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=110

$$-\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^4}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^4+\frac{3}{2}x^2\sinh^{-1}(\cdots$$

```
[Out] (3*x^2)/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + (3*ArcSinh[a*x]^2)
/(4*a^2) + (3*x^2*ArcSinh[a*x]^2)/2 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/
a + ArcSinh[a*x]^4/(4*a^2) + (x^2*ArcSinh[a*x]^4)/2
```

Rubi [A] time = 0.239413, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5661, 5758, 5675, 30}

$$-\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^4}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^4+\frac{3}{2}x^2\sinh^{-1}(\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^4,x]
```

```
[Out] (3*x^2)/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + (3*ArcSinh[a*x]^2)
/(4*a^2) + (3*x^2*ArcSinh[a*x]^2)/2 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/
a + ArcSinh[a*x]^4/(4*a^2) + (x^2*ArcSinh[a*x]^4)/2
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sinh^{-1}(ax)^4\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax)^4-(2a)\int\frac{x^2\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+\frac{1}{2}x^2\sinh^{-1}(ax)^4+3\int x\sinh^{-1}(ax)^2\,dx+\frac{\int\frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{a}\\
&= \frac{3}{2}x^2\sinh^{-1}(ax)^2-\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+\frac{\sinh^{-1}(ax)^4}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^4-(3a)\int\frac{x^2\sinh^{-1}(}{\sqrt{1+a^2x}}\\
&= -\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a}+\frac{3}{2}x^2\sinh^{-1}(ax)^2-\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+\frac{\sinh^{-1}(ax)^4}{4a^2}+\frac{1}{2}x^2\sinh\\
&= \frac{3x^2}{4}-\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a}+\frac{3\sinh^{-1}(ax)^2}{4a^2}+\frac{3}{2}x^2\sinh^{-1}(ax)^2-\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+\frac{\mathrm{si}}{}\end{aligned}$$

Mathematica [A] time = 0.0454885, size = 94, normalized size = 0.85

$$\frac{3a^2x^2+\left(2a^2x^2+1\right)\sinh^{-1}(ax)^4-4ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3+\left(6a^2x^2+3\right)\sinh^{-1}(ax)^2-6ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*ArcSinh
[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + (1 + 2*a^2*x^2)*ArcSinh[
a*x]^4)/(4*a^2)
```

Maple [A] time = 0.026, size = 105, normalized size = 1.

$$\frac{1}{a^{2}}\left(\frac{\left(\operatorname{Arcsinh}\left(ax\right)\right)^{4}\left(a^{2}x^{2}+1\right)}{2}-\left(\operatorname{Arcsinh}\left(ax\right)\right)^{3}ax\sqrt{a^{2}x^{2}+1}-\frac{\left(\operatorname{Arcsinh}\left(ax\right)\right)^{4}}{4}+\frac{3\left(\operatorname{Arcsinh}\left(ax\right)\right)^{2}\left(a^{2}x^{2}+1\right)}{2}-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^2*(1/2*arcsinh(a*x)^4*(a^2*x^2+1)-arcsinh(a*x)^3*a*x*(a^2*x^2+1)^(1/2)-
1/4*arcsinh(a*x)^4+3/2*arcsinh(a*x)^2*(a^2*x^2+1)-3/2*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)*a*x-3/4*arcsinh(a*x)^2+3/4*a^2*x^2+3/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^{2}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{4}-\int\frac{2\left(a^{3}x^{4}+\sqrt{a^{2}x^{2}+1}a^{2}x^{3}+ax^{2}\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}}{a^{3}x^{3}+ax+\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2*(a^3*x^4 + sqrt(a^2*x^
2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^
2*x^2 + 1)^(3/2)), x)
```

Fricas [A] time = 2.05094, size = 316, normalized size = 2.87

$$-\frac{4\sqrt{a^{2}x^{2}+1}ax\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}-\left(2a^{2}x^{2}+1\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{4}-3a^{2}x^{2}+6\sqrt{a^{2}x^{2}+1}ax\log\left(ax+\sqrt{a^{2}x^{2}\right.}{4a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - (2*a^2*x^2 +
 1)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 3*a^2*x^2 + 6*sqrt(a^2*x^2 + 1)*a*x*lo
g(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
^2)/a^2
```

Sympy [A] time = 2.14062, size = 104, normalized size = 0.95

$$\begin{cases} \frac{x^2 \operatorname{asinh}^4(ax)}{2} + \frac{3x^2 \operatorname{asinh}^2(ax)}{2} + \frac{3x^2}{4} - \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a} + \frac{\operatorname{asinh}^4(ax)}{4a^2} + \frac{3\operatorname{asinh}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**4/2 + 3*x**2*asinh(a*x)**2/2 + 3*x**2/4 - x*sqr
t(a**2*x**2 + 1)*asinh(a*x)**3/a - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a)
 + asinh(a*x)**4/(4*a**2) + 3*asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^4, x)
```

3.37 $\int \sinh^{-1}(ax)^4\,dx$

Optimal. Leaf size=67

$$-\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a}-\frac{24\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^4+12x\sinh^{-1}(ax)^2+24x$$

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Rubi [A] time = 0.125115, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5653, 5717, 8}

$$-\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a}-\frac{24\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^4+12x\sinh^{-1}(ax)^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(ax)^4\,dx &= x\sinh^{-1}(ax)^4-(4a)\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4+12\int \sinh^{-1}(ax)^2\,dx\\
&= 12x\sinh^{-1}(ax)^2-\frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4-(24a)\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{24\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a}+12x\sinh^{-1}(ax)^2-\frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4+24\int 1\,dx\\
&= 24x-\frac{24\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a}+12x\sinh^{-1}(ax)^2-\frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4
\end{aligned}$$

Mathematica [A] time = 0.018935, size = 67, normalized size = 1.

$$-\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a}-\frac{24\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^4+12x\sinh^{-1}(ax)^2+24x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Maple [A] time = 0.026, size = 65, normalized size = 1.

$$\frac{1}{a}\left((\mathrm{Arcsinh}\,(ax))^4\,ax-4\,(\mathrm{Arcsinh}\,(ax))^3\sqrt{a^2x^2+1}+12\,(\mathrm{Arcsinh}\,(ax))^2\,ax-24\,\mathrm{Arcsinh}\,(ax)\sqrt{a^2x^2+1}+24\,ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4,x)
```

```
[Out] 1/a*(arcsinh(a*x)^4*a*x-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+12*arcsinh(a*x)^
2*a*x-24*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+24*a*x)
```

Maxima [A] time = 1.13815, size = 99, normalized size = 1.48

$$x \operatorname{arsinh}(ax)^4 - \frac{4\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^3}{a} + 12\left(\frac{x\operatorname{arsinh}(ax)^2}{a} + \frac{2\left(x - \frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a}\right)}{a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^4 - 4*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a + 12*(x*arcsinh(a*x
)^2/a + 2*(x - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a)/a)*a
```

Fricas [A] time = 2.05502, size = 262, normalized size = 3.91

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 12\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 4\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 24\,ax - 24\sqrt{a^2x^2+1}\,\mathrm{lo}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^4 + 12*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2
 - 4*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 24*a*x - 24*sqrt(a^
2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] time = 1.03254, size = 65, normalized size = 0.97

$$\begin{cases} x\operatorname{asinh}^4(ax) + 12x\operatorname{asinh}^2(ax) + 24x - \frac{4\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a} - \frac{24\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4,x)
```

```
[Out] Piecewise((x*asinh(a*x)**4 + 12*x*asinh(a*x)**2 + 24*x - 4*sqrt(a**2*x**2 +
 1)*asinh(a*x)**3/a - 24*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a, 0)), (0, T
rue))
```

Giac [A] time = 1.58945, size = 169, normalized size = 2.52

$$x\log\left(ax+\sqrt{a^2x^2+1}\right)^4-4\left(\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2}-\frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x\right.}}{a^2}\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^4 - 4*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*
x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2
*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2))/a)*a
```

3.38 $\int \frac{\sinh^{-1}(ax)^4}{x}\,dx$

Optimal. Leaf size=97

$$2\sinh^{-1}(ax)^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-3\sinh^{-1}(ax)^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)\text{PolyLog}\left(4,e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -ArcSinh[a*x]^5/5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSinh[
a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*Ar
cSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog[5
, E^(2*ArcSinh[a*x])])/2
```

Rubi [A] time = 0.122107, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$2\sinh^{-1}(ax)^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-3\sinh^{-1}(ax)^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)\text{PolyLog}\left(4,e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x,x]
```

```
[Out] -ArcSinh[a*x]^5/5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSinh[
a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*Ar
cSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog[5
, E^(2*ArcSinh[a*x])])/2
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^4}{x}\,dx &= \text{Subst}\left(\int x^4\coth(x)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^4}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) - 4\,\text{Subst}\left(\int x^3\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}(ax\right.\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 6\,\text{Subst}\left(\int\right.\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\mathbf{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax
\end{aligned}
$$

Mathematica [A] time = 0.0077906, size = 97, normalized size = 1.

$$2\sinh^{-1}(ax)^3\mathbf{PolyLog}\left(2, e^{2\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\mathbf{PolyLog}\left(3, e^{2\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)\mathbf{PolyLog}\left(4, e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^4/x,x]
```

```
[Out] -ArcSinh[a*x]^5/5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSinh[
a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*Ar
cSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog[5
, E^(2*ArcSinh[a*x])])/2
```

Maple [A] time = 0.036, size = 257, normalized size = 2.7

$$-\frac{(\text{Arcsinh}\,(ax))^5}{5} + (\text{Arcsinh}\,(ax))^4\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 4\,(\text{Arcsinh}\,(ax))^3\,\text{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right) - 12\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x,x)
```

```
[Out] -1/5*arcsinh(a*x)^5+arcsinh(a*x)^4*ln(1+a*x+(a^2*x^2+1)^(1/2))+4*arcsinh(a*
x)^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-12*arcsinh(a*x)^2*polylog(3,-a*x-(a^
2*x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(4,-a*x-(a^2*x^2+1)^(1/2))-24*polylo
g(5,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^4*ln(1-a*x-(a^2*x^2+1)^(1/2))+4*ar
csinh(a*x)^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))-12*arcsinh(a*x)^2*polylog(3,a
*x+(a^2*x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(4,a*x+(a^2*x^2+1)^(1/2))-24*p
olylog(5,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^4/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(ax)^4}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^4(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x,x)
```

```
[Out] Integral(asinh(a*x)**4/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x, x)
```

3.39 $\int \frac{\sinh^{-1}(ax)^4}{x^2}\,dx$

Optimal. Leaf size=120

$$-12a\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+12a\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+24a\sinh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -(ArcSinh[a*x]^4/x) - 8*a*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 12*a*Arc
Sinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 12*a*ArcSinh[a*x]^2*PolyLog[2, E^
ArcSinh[a*x]] + 24*a*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] - 24*a*ArcSin
h[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 24*a*PolyLog[4, -E^ArcSinh[a*x]] + 24*a
*PolyLog[4, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.188349, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5661, 5760, 4182, 2531, 6609, 2282, 6589}

$$-12a\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+12a\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+24a\sinh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x^2,x]
```


```
[Out] -(ArcSinh[a*x]^4/x) - 8*a*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 12*a*Arc
Sinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 12*a*ArcSinh[a*x]^2*PolyLog[2, E^
ArcSinh[a*x]] + 24*a*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] - 24*a*ArcSin
h[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 24*a*PolyLog[4, -E^ArcSinh[a*x]] + 24*a
*PolyLog[4, E^ArcSinh[a*x]]
```


Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
```

```
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^4}{x^2}\,dx = -\frac{\sinh^{-1}(ax)^4}{x} + (4a)\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\sinh^{-1}(ax)^4}{x} + (4a)\,\mathrm{Subst}\left(\int x^3\mathbf{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (12a)\,\mathrm{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(a$$
$$= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\sinh^{-1}$$
$$= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\sinh^{-1}$$
$$= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\sinh^{-1}$$
$$= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\sinh^{-1}$$

Mathematica [A] time = 0.226236, size = 161, normalized size = 1.34

$$\frac{1}{2}a\left(24\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) + 24\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right) + 48\sinh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{-s}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^4/x^2,x]
```

```
[Out] (a*(Pi^4 - 2*ArcSinh[a*x]^4 - (2*ArcSinh[a*x]^4)/(a*x) - 8*ArcSinh[a*x]^3*L
og[1 + E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*A
rcSinh[a*x]^2*PolyLog[2, -E^(-ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2,
 E^ArcSinh[a*x]] + 48*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcS
inh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 4
8*PolyLog[4, E^ArcSinh[a*x]]))/2
```

Maple [A] time = 0.05, size = 217, normalized size = 1.8

$$-\frac{(\mathrm{Arcsinh}\,(ax))^4}{x} - 4\,a\,(\mathrm{Arcsinh}\,(ax))^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 12\,a\,(\mathrm{Arcsinh}\,(ax))^2\,\mathrm{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x^2,x)
```

```
[Out] -arcsinh(a*x)^4/x-4*a*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-12*a*arcsi
nh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+24*a*arcsinh(a*x)*polylog(3,-a*
x-(a^2*x^2+1)^(1/2))-24*a*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+4*a*arcsinh(a*x
)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+12*a*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+
1)^(1/2))-24*a*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+24*a*polylog(4
,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{x}+\int\frac{4\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^3x^4+ax^2+\left(a^2x^3+x\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^4/x + integrate(4*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^4}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}^4(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x**2,x)
```

```
[Out] Integral(asinh(a*x)**4/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x^2, x)
```

3.40 $\int \frac{\sinh^{-1}(ax)^4}{x^3}\,dx$

Optimal. Leaf size=108

$$6a^2\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-3a^2\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{2a\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{x}-2a^2\sinh^{-1}(ax)^3+$$

```
[Out] -2*a^2*ArcSinh[a*x]^3 - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x - ArcSinh[
a*x]^4/(2*x^2) + 6*a^2*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 6*a^2*A
rcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*a^2*PolyLog[3, E^(2*ArcSinh[
a*x])]
```

Rubi [A] time = 0.20714, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5661, 5723, 5659, 3716, 2190, 2531, 2282, 6589}

$$6a^2\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-3a^2\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{2a\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{x}-2a^2\sinh^{-1}(ax)^3+$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x^3,x]
```

```
[Out] -2*a^2*ArcSinh[a*x]^3 - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x - ArcSinh[
a*x]^4/(2*x^2) + 6*a^2*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 6*a^2*A
rcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*a^2*PolyLog[3, E^(2*ArcSinh[
a*x])]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
```

```
c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^4}{x^3}\,dx &= -\frac{\sinh^{-1}(ax)^4}{2x^2} + (2a)\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + \left(6a^2\right)\int \frac{\sinh^{-1}(ax)^2}{x}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + \left(6a^2\right)\mathrm{Subst}\left(\int x^2\coth(x)\,dx, x, \sinh^{-1}(ax)\right) \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} - \left(12a^2\right)\mathrm{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \sin\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(a}\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(a}\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(a}\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(a}\right.
\end{aligned}
$$

Mathematica [C] time = 0.259094, size = 113, normalized size = 1.05

$$
-\frac{\sinh^{-1}(ax)^4}{2x^2} + \frac{1}{4}a^2\left(24\sinh^{-1}(ax)\mathrm{PolyLog}\left(2, e^{2\sinh^{-1}(ax)}\right) - 12\mathrm{PolyLog}\left(3, e^{2\sinh^{-1}(ax)}\right) - \frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{ax} - \right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^4/x^3,x]
```

```
[Out] -ArcSinh[a*x]^4/(2*x^2) + (a^2*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^3)/(a*x) + 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])]
 + 24*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcS
inh[a*x])]))/4
```

Maple [A] time = 0.068, size = 208, normalized size = 1.9

$$-2\,a^2\left(\operatorname{Arcsinh}(ax)\right)^3-2\,\frac{a\left(\operatorname{Arcsinh}(ax)\right)^3\sqrt{a^2x^2+1}}{x}-\frac{\left(\operatorname{Arcsinh}(ax)\right)^4}{2\,x^2}+6\,a^2\left(\operatorname{Arcsinh}(ax)\right)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x^3,x)
```

```
[Out] -2*a^2*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x-1/2*arcsinh(a*
x)^4/x^2+6*a^2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+12*a^2*arcsinh(a*
x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-12*a^2*polylog(3,-a*x-(a^2*x^2+1)^(1/2
))+6*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+12*a^2*arcsinh(a*x)*pol
ylog(2,a*x+(a^2*x^2+1)^(1/2))-12*a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{2\,x^2}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^3x^5+ax^3+\left(a^2x^4+x^2\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a^2*x^2 + 1))^4/x^2 + integrate(2*(a^3*x^2 + sqrt(a^2*x
^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^5 + a*x^3 + (a^2*x
^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^4}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^4(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x**3,x)
```

```
[Out] Integral(asinh(a*x)**4/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x^3, x)
```

3.41 $\int \frac{\sinh^{-1}(ax)^4}{x^4}\,dx$

Optimal. Leaf size=223

$$2a^3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-2a^3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-4a^3\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}\right.$$

```
[Out] (-2*a^2*ArcSinh[a*x]^2)/x - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*x^2)
- ArcSinh[a*x]^4/(3*x^3) - 8*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + (4*
a^3*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]])/3 - 4*a^3*PolyLog[2, -E^ArcSinh
[a*x]] + 2*a^3*ArcSinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 4*a^3*PolyLog[2
, E^ArcSinh[a*x]] - 2*a^3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] - 4*a^3
*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] + 4*a^3*ArcSinh[a*x]*PolyLog[3, E
^ArcSinh[a*x]] + 4*a^3*PolyLog[4, -E^ArcSinh[a*x]] - 4*a^3*PolyLog[4, E^Arc
Sinh[a*x]]
```

Rubi [A] time = 0.393155, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5661, 5747, 5760, 4182, 2531, 6609, 2282, 6589, 2279, 2391}

$$2a^3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-2a^3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-4a^3\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x^4,x]
```

```
[Out] (-2*a^2*ArcSinh[a*x]^2)/x - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*x^2)
- ArcSinh[a*x]^4/(3*x^3) - 8*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + (4*
a^3*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]])/3 - 4*a^3*PolyLog[2, -E^ArcSinh
[a*x]] + 2*a^3*ArcSinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 4*a^3*PolyLog[2
, E^ArcSinh[a*x]] - 2*a^3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] - 4*a^3
*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] + 4*a^3*ArcSinh[a*x]*PolyLog[3, E
^ArcSinh[a*x]] + 4*a^3*PolyLog[4, -E^ArcSinh[a*x]] - 4*a^3*PolyLog[4, E^Arc
Sinh[a*x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^4}{x^4}\,dx = -\frac{\sinh^{-1}(ax)^4}{3x^3} + \frac{1}{3}(4a)\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} + \left(2a^2\right)\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx - \frac{1}{3}\left(2a^3\right)\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - \frac{1}{3}\left(2a^3\right)\mathrm{Subst}\left(\int x^3\mathbf{csch}(x)\,dx, x\right.$$

$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} + \frac{4}{3}a^3\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax}\right.$$

$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Mathematica [A] time = 2.53809, size = 355, normalized size = 1.59

$$\frac{1}{24}a^3\left(-48\sinh^{-1}(ax)^2\mathbf{PolyLog}\left(2, e^{\sinh^{-1}(ax)}\right) - 96\sinh^{-1}(ax)\mathbf{PolyLog}\left(3, -e^{-\sinh^{-1}(ax)}\right) + 96\sinh^{-1}(ax)\mathbf{PolyLog}\left(3, \right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^4/x^4,x]
```

```
[Out] (a^3*(-2*Pi^4 + 4*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*Coth[ArcSinh[a*x]/2] +
 2*ArcSinh[a*x]^4*Coth[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]
/2]^2 - (a*x*ArcSinh[a*x]^4*Csch[ArcSinh[a*x]/2]^4)/2 + 96*ArcSinh[a*x]*Log
[1 - E^(-ArcSinh[a*x])] - 96*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + 16*A
rcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 16*ArcSinh[a*x]^3*Log[1 - E^ArcS
inh[a*x]] - 48*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-ArcSinh[a*x])] - 96*Po
lyLog[2, E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]]
- 96*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] + 96*ArcSinh[a*x]*PolyLog[
3, E^ArcSinh[a*x]] - 96*PolyLog[4, -E^(-ArcSinh[a*x])] - 96*PolyLog[4, E^Ar
cSinh[a*x]] - 4*ArcSinh[a*x]^3*Sech[ArcSinh[a*x]/2]^2 - (8*ArcSinh[a*x]^4*S
inh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]^2*Tanh[ArcSinh[a*x]/2] -
```

```
2*ArcSinh[a*x]^4*Tanh[ArcSinh[a*x]/2]))/24
```

Maple [A] time = 0.091, size = 372, normalized size = 1.7

$$-\frac{2\,a\left(\mathrm{Arcsinh}\left(ax\right)\right)^{3}}{3\,x^{2}}\sqrt{a^{2}x^{2}+1}-2\,\frac{a^{2}\left(\mathrm{Arcsinh}\left(ax\right)\right)^{2}}{x}-\frac{\left(\mathrm{Arcsinh}\left(ax\right)\right)^{4}}{3\,x^{3}}+\frac{2\,a^{3}\left(\mathrm{Arcsinh}\left(ax\right)\right)^{3}}{3}\ln\left(1+ax+\sqrt{a^{2}x^{2}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x^4,x)
```

```
[Out] -2/3*a*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x^2-2*a^2*arcsinh(a*x)^2/x-1/3*arcs
inh(a*x)^4/x^3+2/3*a^3*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*a^3*arc
sinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-4*a^3*arcsinh(a*x)*polylog(3,
-a*x-(a^2*x^2+1)^(1/2))+4*a^3*polylog(4,-a*x-(a^2*x^2+1)^(1/2))-2/3*a^3*arc
sinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-2*a^3*arcsinh(a*x)^2*polylog(2,a*x+
(a^2*x^2+1)^(1/2))+4*a^3*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))-4*a^
3*polylog(4,a*x+(a^2*x^2+1)^(1/2))-4*a^3*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^
(1/2))-4*a^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+4*a^3*arcsinh(a*x)*ln(1-a*x-
(a^2*x^2+1)^(1/2))+4*a^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{4}}{3\,x^{3}}+\int\frac{4\left(a^{3}x^{2}+\sqrt{a^{2}x^{2}+1}a^{2}x+a\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}}{3\left(a^{3}x^{6}+ax^{4}+\left(a^{2}x^{5}+x^{3}\right)\sqrt{a^{2}x^{2}+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^4/x^3 + integrate(4/3*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^6 + a*x^4 + (a^2
*x^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}\left(ax\right)^{4}}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^4(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x**4,x)
```

```
[Out] Integral(asinh(a*x)**4/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x^4, x)
```

3.42 $\int \frac{x^6}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$-\frac{5\text{Chi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\text{Chi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{\text{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]])/(64*a^7) + (9*CoshIntegral[3*ArcSinh[a*x]])
/(64*a^7) - (5*CoshIntegral[5*ArcSinh[a*x]])/(64*a^7) + CoshIntegral[7*ArcS
inh[a*x]]/(64*a^7)
```

Rubi [A] time = 0.100261, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5669, 5448, 3301}

$$-\frac{5\text{Chi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\text{Chi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{\text{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcSinh[a*x],x]
```

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]])/(64*a^7) + (9*CoshIntegral[3*ArcSinh[a*x]])
/(64*a^7) - (5*CoshIntegral[5*ArcSinh[a*x]])/(64*a^7) + CoshIntegral[7*ArcS
inh[a*x]]/(64*a^7)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^7} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64x}+\frac{9\cosh(3x)}{64x}-\frac{5\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right) dx, x, \sinh^{-1}(ax)\right)}{a^7} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a^7} - \frac{5\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a^7} - \frac{5\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,d\right.}{64a^7} \\
&= -\frac{5\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{9\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\textbf{Chi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{\textbf{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}
\end{aligned}$$

Mathematica [A] time = 0.0118237, size = 40, normalized size = 0.73

$$\frac{-5\textbf{Chi}\left(\sinh^{-1}(ax)\right)+9\textbf{Chi}\left(3\sinh^{-1}(ax)\right)-5\textbf{Chi}\left(5\sinh^{-1}(ax)\right)+\textbf{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcSinh[a*x],x]
```

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]] + 9*CoshIntegral[3*ArcSinh[a*x]] - 5*CoshInt
egral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Maple [A] time = 0.041, size = 40, normalized size = 0.7

$$\frac{1}{a^7}\left(-\frac{5\,\text{Chi}\,(\text{Arcsinh}\,(ax))}{64}+\frac{9\,\text{Chi}\,(3\,\text{Arcsinh}\,(ax))}{64}-\frac{5\,\text{Chi}\,(5\,\text{Arcsinh}\,(ax))}{64}+\frac{\text{Chi}\,(7\,\text{Arcsinh}\,(ax))}{64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arcsinh(a*x),x)
```

```
[Out] 1/a^7*(-5/64*Chi(arcsinh(a*x))+9/64*Chi(3*arcsinh(a*x))-5/64*Chi(5*arcsinh(
a*x))+1/64*Chi(7*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/asinh(a*x),x)
```

```
[Out] Integral(x**6/asinh(a*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{arsinh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/arcsinh(a*x), x)
```

3.43 $\int \frac{x^5}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}\left(2\sinh^{-1}(ax)\right)}{32a^6}-\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^6}+\frac{\text{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]])/(32*a^6) - SinhIntegral[4*ArcSinh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcSinh[a*x]]/(32*a^6)
```

Rubi [A] time = 0.0826723, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5669, 5448, 3298}

$$\frac{5\text{Shi}\left(2\sinh^{-1}(ax)\right)}{32a^6}-\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^6}+\frac{\text{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcSinh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]])/(32*a^6) - SinhIntegral[4*ArcSinh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcSinh[a*x]]/(32*a^6)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^5}{\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} - \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{8a^6} + \frac{5\,\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx\right.}{32a^6}$$
$$= \frac{5\text{Shi}\left(2\sinh^{-1}(ax)\right)}{32a^6} - \frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

Mathematica [A] time = 0.110397, size = 33, normalized size = 0.77

$$\frac{5\text{Shi}\left(2\sinh^{-1}(ax)\right) - 4\text{Shi}\left(4\sinh^{-1}(ax)\right) + \text{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcSinh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]] - 4*SinhIntegral[4*ArcSinh[a*x]] + SinhInte
gral[6*ArcSinh[a*x]])/(32*a^6)
```

Maple [A] time = 0.033, size = 33, normalized size = 0.8

$$\frac{1}{a^6}\left(\frac{5\,\text{Shi}\,(2\,\text{Arcsinh}\,(ax))}{32} - \frac{\text{Shi}\,(4\,\text{Arcsinh}\,(ax))}{8} + \frac{\text{Shi}\,(6\,\text{Arcsinh}\,(ax))}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arcsinh(a*x),x)
```

```
[Out] 1/a^6*(5/32*Shi(2*arcsinh(a*x))-1/8*Shi(4*arcsinh(a*x))+1/32*Shi(6*arcsinh(
a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/asinh(a*x),x)
```

```
[Out] Integral(x**5/asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/arcsinh(a*x), x)
```

3.44 $\int \frac{x^4}{\sinh^{-1}(ax)} dx$

Optimal. Leaf size=41

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^5} - \frac{3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

```
[Out] CoshIntegral[ArcSinh[a*x]]/(8*a^5) - (3*CoshIntegral[3*ArcSinh[a*x]])/(16*a
^5) + CoshIntegral[5*ArcSinh[a*x]]/(16*a^5)
```

Rubi [A] time = 0.0837777, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5669, 5448, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^5} - \frac{3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x],x]
```

```
[Out] CoshIntegral[ArcSinh[a*x]]/(8*a^5) - (3*CoshIntegral[3*ArcSinh[a*x]])/(16*a
^5) + CoshIntegral[5*ArcSinh[a*x]]/(16*a^5)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x} - \frac{3\cosh(3x)}{16x} + \frac{\cosh(5x)}{16x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx\right.}{16a^5}\\
&= \frac{\mathbf{Chi}\left(\sinh^{-1}(ax)\right)}{8a^5} - \frac{3\mathbf{Chi}\left(3\sinh^{-1}(ax)\right)}{16a^5} + \frac{\mathbf{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}
\end{aligned}
$$

Mathematica [A] time = 0.0098321, size = 31, normalized size = 0.76

$$\frac{2\mathbf{Chi}\left(\sinh^{-1}(ax)\right) - 3\mathbf{Chi}\left(3\sinh^{-1}(ax)\right) + \mathbf{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x],x]
```

```
[Out] (2*CoshIntegral[ArcSinh[a*x]] - 3*CoshIntegral[3*ArcSinh[a*x]] + CoshIntegr
al[5*ArcSinh[a*x]])/(16*a^5)
```

Maple [A] time = 0.025, size = 31, normalized size = 0.8

$$\frac{1}{a^5}\left(\frac{\text{Chi}\,(\text{Arcsinh}\,(ax))}{8} - \frac{3\,\text{Chi}\,(3\,\text{Arcsinh}\,(ax))}{16} + \frac{\text{Chi}\,(5\,\text{Arcsinh}\,(ax))}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x),x)
```

```
[Out] 1/a^5*(1/8*Chi(arcsinh(a*x))-3/16*Chi(3*arcsinh(a*x))+1/16*Chi(5*arcsinh(a*
x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x),x)
```

```
[Out] Integral(x**4/asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x), x)
```

3.45 $\int \frac{x^3}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^4}-\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{4a^4}$$

```
[Out] -SinhIntegral[2*ArcSinh[a*x]]/(4*a^4) + SinhIntegral[4*ArcSinh[a*x]]/(8*a^4
)
```

Rubi [A] time = 0.0679608, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5669, 5448, 3298}

$$\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^4}-\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x],x]
```

```
[Out] -SinhIntegral[2*ArcSinh[a*x]]/(4*a^4) + SinhIntegral[4*ArcSinh[a*x]]/(8*a^4
)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{4a^4} + \frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^4}$$

Mathematica [A] time = 0.0717786, size = 24, normalized size = 0.83

$$\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right) - 2\text{Shi}\left(2\sinh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x],x]
```

```
[Out] (-2*SinhIntegral[2*ArcSinh[a*x]] + SinhIntegral[4*ArcSinh[a*x]])/(8*a^4)
```

Maple [A] time = 0.024, size = 24, normalized size = 0.8

$$\frac{1}{a^4}\left(-\frac{\text{Shi}\,(2\,\text{Arcsinh}\,(ax))}{4} + \frac{\text{Shi}\,(4\,\text{Arcsinh}\,(ax))}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x),x)
```

```
[Out] 1/a^4*(-1/4*Shi(2*arcsinh(a*x))+1/8*Shi(4*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\operatorname{arsinh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x),x)
```

```
[Out] Integral(x**3/asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x), x)
```

3.46 $\int \frac{x^2}{\sinh^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}$$

```
[Out] -CoshIntegral[ArcSinh[a*x]]/(4*a^3) + CoshIntegral[3*ArcSinh[a*x]]/(4*a^3)
```

Rubi [A] time = 0.0667451, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5669, 5448, 3301}

$$\frac{\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x],x]
```

```
[Out] -CoshIntegral[ArcSinh[a*x]]/(4*a^3) + CoshIntegral[3*ArcSinh[a*x]]/(4*a^3)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}$$
$$= -\frac{\mathbf{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3} + \frac{\mathbf{Chi}\left(3\sinh^{-1}(ax)\right)}{4a^3}$$

Mathematica [A] time = 0.0064793, size = 22, normalized size = 0.81

$$\frac{\mathbf{Chi}\left(3\sinh^{-1}(ax)\right) - \mathbf{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x],x]
```

```
[Out] (-CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Maple [A] time = 0.02, size = 22, normalized size = 0.8

$$\frac{1}{a^3}\left(-\frac{\text{Chi}\,(\text{Arcsinh}\,(ax))}{4} + \frac{\text{Chi}\,(3\,\text{Arcsinh}\,(ax))}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x),x)
```

```
[Out] 1/a^3*(-1/4*Chi(arcsinh(a*x))+1/4*Chi(3*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^2}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x),x)
```

```
[Out] Integral(x**2/asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x), x)
```

3.47 $\int \frac{x}{\sinh^{-1}(ax)} dx$

Optimal. Leaf size=14

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}$$

[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)

Rubi [A] time = 0.0379933, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5669, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSinh[a*x],x]

[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\ &= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\ &= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}\\ &= \frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0202518, size = 14, normalized size = 1.

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x],x]
```

```
[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

Maple [A] time = 0.03, size = 13, normalized size = 0.9

$$\frac{\text{Shi}\,(2\,\text{Arcsinh}\,(ax))}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x),x)
```

```
[Out] 1/2*Shi(2*arcsinh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x),x)
```

```
[Out] Integral(x/asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x), x)
```

3.48 $\int \frac{1}{\sinh^{-1}(ax)} \, dx$

Optimal. Leaf size=9

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

```
[Out] CoshIntegral[ArcSinh[a*x]]/a
```

Rubi [A] time = 0.017505, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5657, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a*x]]/a
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.0089621, size = 9, normalized size = 1.

$$\frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a*x]]/a
```

Maple [A] time = 0.019, size = 10, normalized size = 1.1

$$\frac{\text{Chi}\left(\text{Arcsinh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x),x)
```

```
[Out] Chi(arcsinh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\text{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x),x)
```

```
[Out] Integral(1/asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/arcsinh(a*x), x)
```

3.49 $\int \frac{1}{x \sinh^{-1}(ax)} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]), x]
```

Rubi [A] time = 0.0140428, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)} dx = \int \frac{1}{x \sinh^{-1}(ax)} dx$$

Mathematica [A] time = 0.171755, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]), x]
```

Maple [A] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{1}{x\mathrm{Arcsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x),x)
```

```
[Out] int(1/x/arcsinh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\operatorname{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x\operatorname{arsinh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\operatorname{asinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x),x)
```

```
[Out] Integral(1/(x*asinh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)), x)
```

3.50 $\int \frac{1}{x^2 \sinh^{-1}(ax)}\, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \sinh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcSinh[a*x]), x]
```

Rubi [A] time = 0.0149415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)}\, dx = \int \frac{1}{x^2 \sinh^{-1}(ax)}\, dx$$

Mathematica [A] time = 0.803094, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sinh^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*ArcSinh[a*x]), x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \mathrm{Arcsinh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x),x)
```

```
[Out] int(1/x^2/arcsinh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \operatorname{arsinh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^{2} \operatorname{arsinh}\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arcsinh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \operatorname{asinh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x),x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)), x)
```

3.51 $\int \frac{x^6}{\sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=82

$$-\frac{5\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{25\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{7\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a^7}-\frac{x^6\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -((x^6*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - (5*SinhIntegral[ArcSinh[a*x]]
)/(64*a^7) + (27*SinhIntegral[3*ArcSinh[a*x]])/(64*a^7) - (25*SinhIntegral[
5*ArcSinh[a*x]])/(64*a^7) + (7*SinhIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Rubi [A] time = 0.0811772, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5665, 3298}

$$-\frac{5\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{25\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{7\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a^7}-\frac{x^6\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^6*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - (5*SinhIntegral[ArcSinh[a*x]]
)/(64*a^7) + (27*SinhIntegral[3*ArcSinh[a*x]])/(64*a^7) - (25*SinhIntegral[
5*ArcSinh[a*x]])/(64*a^7) + (7*SinhIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^6}{\sinh^{-1}(ax)^2}\,dx = -\frac{x^6\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{5\sinh(x)}{64x}+\frac{27\sinh(3x)}{64x}-\frac{25\sinh(5x)}{64x}+\frac{7\sinh(7x)}{64x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^7}$$

$$= -\frac{x^6\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} - \frac{5\,\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a^7} + \frac{7\,\text{Subst}\left(\int\frac{\sinh(7x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a^7} - \frac{25}{}$$

$$= -\frac{x^6\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} - \frac{5\mathbf{Shi}\left(\sinh^{-1}(ax)\right)}{64a^7} + \frac{27\mathbf{Shi}\left(3\sinh^{-1}(ax)\right)}{64a^7} - \frac{25\mathbf{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7} + \frac{7\mathbf{Shi}\left(7\sinh\right.}{64a^7}$$

Mathematica [A] time = 0.279956, size = 85, normalized size = 1.04

$$-\frac{64a^6x^6\sqrt{a^2x^2+1}+5\sinh^{-1}(ax)\mathbf{Shi}\left(\sinh^{-1}(ax)\right)-27\sinh^{-1}(ax)\mathbf{Shi}\left(3\sinh^{-1}(ax)\right)+25\sinh^{-1}(ax)\mathbf{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcSinh[a*x]^2,x]
```

```
[Out] -(64*a^6*x^6*Sqrt[1 + a^2*x^2] + 5*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
- 27*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 25*ArcSinh[a*x]*SinhIntegr
al[5*ArcSinh[a*x]] - 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]])/(64*a^7*A
rcSinh[a*x])
```

Maple [A] time = 0.043, size = 104, normalized size = 1.3

$$\frac{1}{a^7}\left(\frac{5}{64\,\text{Arcsinh}\,(ax)}\sqrt{a^2x^2+1} - \frac{5\,\text{Shi}\,(\text{Arcsinh}\,(ax))}{64} - \frac{9\,\cosh\,(3\,\text{Arcsinh}\,(ax))}{64\,\text{Arcsinh}\,(ax)} + \frac{27\,\text{Shi}\,(3\,\text{Arcsinh}\,(ax))}{64} + \frac{5\,\cosh\,(}{64\,\text{A}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^7*(5/64/arcsinh(a*x)*(a^2*x^2+1)^(1/2)-5/64*Shi(arcsinh(a*x))-9/64/arcs
inh(a*x)*cosh(3*arcsinh(a*x))+27/64*Shi(3*arcsinh(a*x))+5/64/arcsinh(a*x)*c
osh(5*arcsinh(a*x))-25/64*Shi(5*arcsinh(a*x))-1/64/arcsinh(a*x)*cosh(7*arcs
inh(a*x))+7/64*Shi(7*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^9 + ax^7 + \left(a^2x^8 + x^6\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{7\,a^5x^{10} + 14\,a^3x^8 + 7\,ax^6 + \left(7\,a^3x^8 + 5\,ax^6\right)\left(a^2x^2+1\right) + \left(14\,a^4}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^9 + a*x^7 + (a^2*x^8 + x^6)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((7*a^5*x^10
+ 14*a^3*x^8 + 7*a*x^6 + (7*a^3*x^8 + 5*a*x^6)*(a^2*x^2 + 1) + (14*a^4*x^9
+ 19*a^2*x^7 + 6*x^5)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
+ 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2
*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/asinh(a*x)**2,x)
```

```
[Out] Integral(x**6/asinh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/arcsinh(a*x)^2, x)
```

3.52 $\int \frac{x^5}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{16a^6}-\frac{\mathrm{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^6}+\frac{3\mathrm{Chi}\left(6\sinh^{-1}(ax)\right)}{16a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -((x^5*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (5*CoshIntegral[2*ArcSinh[a*x
]])/(16*a^6) - CoshIntegral[4*ArcSinh[a*x]]/(2*a^6) + (3*CoshIntegral[6*Arc
Sinh[a*x]])/(16*a^6)
```

Rubi [A] time = 0.0664206, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5665, 3301}

$$\frac{5\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{16a^6}-\frac{\mathrm{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^6}+\frac{3\mathrm{Chi}\left(6\sinh^{-1}(ax)\right)}{16a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^5*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (5*CoshIntegral[2*ArcSinh[a*x
]])/(16*a^6) - CoshIntegral[4*ArcSinh[a*x]]/(2*a^6) + (3*CoshIntegral[6*Arc
Sinh[a*x]])/(16*a^6)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x^5\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(\frac{5\cosh(2x)}{16x} - \frac{\cosh(4x)}{2x} + \frac{3\cosh(6x)}{16x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^6} \\
&= -\frac{x^5\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int\frac{\cosh(6x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a^6} + \frac{5\,\text{Subst}\left(\int\frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a^6} - \cdots \\
&= -\frac{x^5\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{5\mathbf{Chi}\left(2\sinh^{-1}(ax)\right)}{16a^6} - \frac{\mathbf{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^6} + \frac{3\mathbf{Chi}\left(6\sinh^{-1}(ax)\right)}{16a^6}
\end{aligned}
$$

Mathematica [A] time = 0.0408332, size = 78, normalized size = 1.11

$$
-\frac{-10\sinh^{-1}(ax)\mathbf{Chi}\left(2\sinh^{-1}(ax)\right) + 16\sinh^{-1}(ax)\mathbf{Chi}\left(4\sinh^{-1}(ax)\right) - 6\sinh^{-1}(ax)\mathbf{Chi}\left(6\sinh^{-1}(ax)\right) + 5\sinh\left(2\sin\cdots\right.}{32a^6\sinh^{-1}(ax)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcSinh[a*x]^2,x]
```

```
[Out] -(-10*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] + 16*ArcSinh[a*x]*CoshInteg
ral[4*ArcSinh[a*x]] - 6*ArcSinh[a*x]*CoshIntegral[6*ArcSinh[a*x]] + 5*Sinh[
2*ArcSinh[a*x]] - 4*Sinh[4*ArcSinh[a*x]] + Sinh[6*ArcSinh[a*x]])/(32*a^6*Ar
cSinh[a*x])
```

Maple [A] time = 0.038, size = 78, normalized size = 1.1

$$
\frac{1}{a^6}\left(-\frac{5\sinh(2\,\text{Arcsinh}(ax))}{32\,\text{Arcsinh}(ax)} + \frac{5\,\text{Chi}(2\,\text{Arcsinh}(ax))}{16} + \frac{\sinh(4\,\text{Arcsinh}(ax))}{8\,\text{Arcsinh}(ax)} - \frac{\text{Chi}(4\,\text{Arcsinh}(ax))}{2} - \frac{\sinh(6\,\text{Arcs}\cdots}{32\,\text{Arcsin}\cdots}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^6*(-5/32/arcsinh(a*x)*sinh(2*arcsinh(a*x))+5/16*Chi(2*arcsinh(a*x))+1/8
/arcsinh(a*x)*sinh(4*arcsinh(a*x))-1/2*Chi(4*arcsinh(a*x))-1/32/arcsinh(a*x
)*sinh(6*arcsinh(a*x))+3/16*Chi(6*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^8 + ax^6 + (a^2x^7 + x^5)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{6\,a^5x^9 + 12\,a^3x^7 + 6\,ax^5 + 2\left(3\,a^3x^7 + 2\,ax^5\right)\left(a^2x^2+1\right) + \left(12\,a\right.}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2x^2+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^8 + a*x^6 + (a^2*x^7 + x^5)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((6*a^5*x^9 +
 12*a^3*x^7 + 6*a*x^5 + 2*(3*a^3*x^7 + 2*a*x^5)*(a^2*x^2 + 1) + (12*a^4*x^8
 + 16*a^2*x^6 + 5*x^4)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^
2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5}{\text{arsinh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/asinh(a*x)**2,x)
```

```
[Out] Integral(x**5/asinh(a*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/arcsinh(a*x)^2, x)
```

3.53 $\int \frac{x^4}{\sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=68

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -((x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/(8
*a^5) - (9*SinhIntegral[3*ArcSinh[a*x]])/(16*a^5) + (5*SinhIntegral[5*ArcSi
nh[a*x]])/(16*a^5)
```

Rubi [A] time = 0.0658968, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5665, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/(8
*a^5) - (9*SinhIntegral[3*ArcSinh[a*x]])/(16*a^5) + (5*SinhIntegral[5*ArcSi
nh[a*x]])/(16*a^5)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^2}\,dx = -\frac{x^4\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(\frac{\sinh(x)}{8x}-\frac{9\sinh(3x)}{16x}+\frac{5\sinh(5x)}{16x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}dx, x, \sinh^{-1}(ax)\right)}{8a^5} + \frac{5\,\text{Subst}\left(\int\frac{\sinh(5x)}{x}dx, x, \sinh^{-1}(ax)\right)}{16a^5} - \frac{9\,\text{Su}}{}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a^5} - \frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a^5} + \frac{5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Mathematica [A] time = 0.202217, size = 60, normalized size = 0.88

$$\frac{-\frac{16a^4x^4\sqrt{a^2x^2+1}}{\sinh^{-1}(ax)} + 2\text{Shi}\left(\sinh^{-1}(ax)\right) - 9\text{Shi}\left(3\sinh^{-1}(ax)\right) + 5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x]^2,x]
```

```
[Out] ((-16*a^4*x^4*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + 2*SinhIntegral[ArcSinh[a*x]
] - 9*SinhIntegral[3*ArcSinh[a*x]] + 5*SinhIntegral[5*ArcSinh[a*x]])/(16*a^
5)
```

Maple [A] time = 0.028, size = 80, normalized size = 1.2

$$\frac{1}{a^5}\left(-\frac{1}{8\,\text{Arcsinh}\,(ax)}\sqrt{a^2x^2+1} + \frac{\text{Shi}\,(\text{Arcsinh}\,(ax))}{8} + \frac{3\,\cosh\,(3\,\text{Arcsinh}\,(ax))}{16\,\text{Arcsinh}\,(ax)} - \frac{9\,\text{Shi}\,(3\,\text{Arcsinh}\,(ax))}{16} - \frac{\cosh\,(5\,\text{Ar}}{16\,\text{Arcsi}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^5*(-1/8/arcsinh(a*x)*(a^2*x^2+1)^(1/2)+1/8*Shi(arcsinh(a*x))+3/16/arcsi
nh(a*x)*cosh(3*arcsinh(a*x))-9/16*Shi(3*arcsinh(a*x))-1/16/arcsinh(a*x)*cos
h(5*arcsinh(a*x))+5/16*Shi(5*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^7 + ax^5 + \left(a^2x^6 + x^4\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{5\,a^5x^8 + 10\,a^3x^6 + 5\,ax^4 + \left(5\,a^3x^6 + 3\,ax^4\right)\left(a^2x^2+1\right) + \left(10\,a^4x\right.}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2x^2+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^7 + a*x^5 + (a^2*x^6 + x^4)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^5*x^8 +
 10*a^3*x^6 + 5*a*x^4 + (5*a^3*x^6 + 3*a*x^4)*(a^2*x^2 + 1) + (10*a^4*x^7 +
 13*a^2*x^5 + 4*x^3)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 +
 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*
x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**2,x)
```

```
[Out] Integral(x**4/asinh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^2, x)
```

3.54 $\int \frac{x^3}{\sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=56

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -((x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - CoshIntegral[2*ArcSinh[a*x]]/
(2*a^4) + CoshIntegral[4*ArcSinh[a*x]]/(2*a^4)
```

Rubi [A] time = 0.0494046, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5665, 3301}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - CoshIntegral[2*ArcSinh[a*x]]/
(2*a^4) + CoshIntegral[4*ArcSinh[a*x]]/(2*a^4)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^2}\,dx = -\frac{x^3\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{2x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= -\frac{x^3\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int\frac{\cosh(2x)}{x}dx, x, \sinh^{-1}(ax)\right)}{2a^4} + \frac{\text{Subst}\left(\int\frac{\cosh(4x)}{x}dx, x, \sinh^{-1}(ax)\right)}{2a^4}$$
$$= -\frac{x^3\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} - \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^4} + \frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^4}$$

Mathematica [A] time = 0.0258267, size = 56, normalized size = 1.

$$-\frac{4\sinh^{-1}(ax)\text{Chi}\left(2\sinh^{-1}(ax)\right)-4\sinh^{-1}(ax)\text{Chi}\left(4\sinh^{-1}(ax)\right)-2\sinh\left(2\sinh^{-1}(ax)\right)+\sinh\left(4\sinh^{-1}(ax)\right)}{8a^4\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x]^2,x]
```

```
[Out] -(4*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] - 4*ArcSinh[a*x]*CoshIntegral
[4*ArcSinh[a*x]] - 2*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])/(8*a^4*Ar
cSinh[a*x])
```

Maple [A] time = 0.025, size = 54, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{\sinh(2\,\text{Arcsinh}(ax))}{4\,\text{Arcsinh}(ax)} - \frac{\text{Chi}(2\,\text{Arcsinh}(ax))}{2} - \frac{\sinh(4\,\text{Arcsinh}(ax))}{8\,\text{Arcsinh}(ax)} + \frac{\text{Chi}(4\,\text{Arcsinh}(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4/arcsinh(a*x)*sinh(2*arcsinh(a*x))-1/2*Chi(2*arcsinh(a*x))-1/8/ar
csinh(a*x)*sinh(4*arcsinh(a*x))+1/2*Chi(4*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^6 + ax^4 + \left(a^2x^5 + x^3\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{4\,a^5x^7 + 8\,a^3x^5 + 4\,ax^3 + 2\left(2\,a^3x^5 + ax^3\right)\left(a^2x^2+1\right) + \left(8\,a^4x^6\right.}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2x^2+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^6 + a*x^4 + (a^2*x^5 + x^3)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((4*a^5*x^7 +
 8*a^3*x^5 + 4*a*x^3 + 2*(2*a^3*x^5 + a*x^3)*(a^2*x^2 + 1) + (8*a^4*x^6 + 1
0*a^2*x^4 + 3*x^2)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2
*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^
2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**2,x)
```

```
[Out] Integral(x**3/asinh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x)^2, x)
```

3.55 $\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -((x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - SinhIntegral[ArcSinh[a*x]]/(4
*a^3) + (3*SinhIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Rubi [A] time = 0.0474095, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5665, 3298}

$$-\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - SinhIntegral[ArcSinh[a*x]]/(4
*a^3) + (3*SinhIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4x}+\frac{3\sinh(3x)}{4x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} + \frac{3\,\text{Subst}\left(\int\frac{\sinh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}$$
$$= -\frac{x^2\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} - \frac{\textbf{Shi}\left(\sinh^{-1}(ax)\right)}{4a^3} + \frac{3\textbf{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}$$

Mathematica [A] time = 0.165811, size = 49, normalized size = 0.91

$$-\frac{\frac{4a^2x^2\sqrt{a^2x^2+1}}{\sinh^{-1}(ax)} + \textbf{Shi}\left(\sinh^{-1}(ax)\right) - 3\textbf{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((4*a^2*x^2*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + SinhIntegral[ArcSinh[a*x]] -
 3*SinhIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Maple [A] time = 0.024, size = 56, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{1}{4\,\text{Arcsinh}\,(ax)}\sqrt{a^2x^2+1} - \frac{\text{Shi}\,(\text{Arcsinh}\,(ax))}{4} - \frac{\cosh\,(3\,\text{Arcsinh}\,(ax))}{4\,\text{Arcsinh}\,(ax)} + \frac{3\,\text{Shi}\,(3\,\text{Arcsinh}\,(ax))}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/4/arcsinh(a*x)*(a^2*x^2+1)^(1/2)-1/4*Shi(arcsinh(a*x))-1/4/arcsinh
(a*x)*cosh(3*arcsinh(a*x))+3/4*Shi(3*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^5+ax^3+\left(a^2x^4+x^2\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)} + \int\frac{3\,a^5x^6+6\,a^3x^4+3\,ax^2+\left(3\,a^3x^4+ax^2\right)\left(a^2x^2+1\right)+\left(6\,a^4x^5+7\right.}{\left(a^5x^4+\left(a^2x^2+1\right)a^3x^2+2\,a^3x^2+2\left(a^4x^3+a^2x\right)\sqrt{a^2x^2+1}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^5 + a*x^3 + (a^2*x^4 + x^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((3*a^5*x^6 +
 6*a^3*x^4 + 3*a*x^2 + (3*a^3*x^4 + a*x^2)*(a^2*x^2 + 1) + (6*a^4*x^5 + 7*a
^2*x^3 + 2*x)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*
x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**2,x)
```

```
[Out] Integral(x**2/asinh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^2, x)
```

3.56 $\int \frac{x}{\sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=37

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2} - \frac{x\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + CoshIntegral[2*ArcSinh[a*x]]/a^
2
```

Rubi [A] time = 0.0246061, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5665, 3301}

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2} - \frac{x\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^2,x]
```

```
[Out] -((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + CoshIntegral[2*ArcSinh[a*x]]/a^
2
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(ax)^2}\,dx = -\frac{x\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\mathbf{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0032678, size = 32, normalized size = 0.86

$$\frac{\mathbf{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2} - \frac{\sinh\left(2\sinh^{-1}(ax)\right)}{2a^2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x]^2,x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/a^2 - Sinh[2*ArcSinh[a*x]]/(2*a^2*ArcSinh[a*x]
)
```

Maple [A] time = 0.026, size = 28, normalized size = 0.8

$$\frac{1}{a^2}\left(-\frac{\sinh(2\,\mathrm{Arcsinh}(ax))}{2\,\mathrm{Arcsinh}(ax)} + \mathrm{Chi}(2\,\mathrm{Arcsinh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/2/arcsinh(a*x)*sinh(2*arcsinh(a*x))+Chi(2*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^4 + ax^2 + \left(a^2x^3 + x\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{2\,a^5x^5 + 2\left(a^2x^2+1\right)a^3x^3 + 4\,a^3x^3 + 2\,ax + \left(4\,a^4x^4 + 4\,a^2x}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2x^2+1} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x
^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((2*a^5*x^5 + 2
*(a^2*x^2 + 1)*a^3*x^3 + 4*a^3*x^3 + 2*a*x + (4*a^4*x^4 + 4*a^2*x^2 + 1)*sq
rt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*x^2 + 2*(a^4*x^3
 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\operatorname{arsinh}\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}^{2}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**2,x)
```

```
[Out] Integral(x/asinh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}\left(ax\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^2, x)
```

3.57 $\int \frac{1}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=34

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{a}-\frac{\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/a
```

Rubi [A] time = 0.0833157, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5655, 5779, 3298}

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{a}-\frac{\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/a
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{-1}(ax)^2}\,dx &= -\frac{\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+a\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}\\ &= -\frac{\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\mathbf{Shi}\left(\sinh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0478953, size = 31, normalized size = 0.91

$$\frac{\mathbf{Shi}\left(\sinh^{-1}(ax)\right)-\frac{\sqrt{a^2x^2+1}}{\sinh^{-1}(ax)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-2),x]
```

```
[Out] (-(Sqrt[1 + a^2*x^2]/ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]])/a
```

Maple [A] time = 0.023, size = 30, normalized size = 0.9

$$\frac{1}{a}\left(-\frac{1}{\mathrm{Arcsinh}\,(ax)}\sqrt{a^2x^2+1}+\mathrm{Shi}\,(\mathrm{Arcsinh}\,(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^2,x)
```

```
[Out] 1/a*(-1/arcsinh(a*x)*(a^2*x^2+1)^(1/2)+Shi(arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^3 + ax + \left(a^2x^2 + 1\right)^{\frac{3}{2}}}{\left(a^3x^2 + \sqrt{a^2x^2 + 1}a^2x + a\right)\log\left(ax + \sqrt{a^2x^2 + 1}\right)} + \int \frac{a^4x^4 + 2\,a^2x^2 + \left(a^2x^2 + 1\right)\left(a^2x^2 - 1\right) + \left(2\,a^3x^3 + ax\right)}{\left(a^4x^4 + \left(a^2x^2 + 1\right)a^2x^2 + 2\,a^2x^2 + 2\left(a^3x^3 + ax\right)\sqrt{a^2x^2 + 1} + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x
+ a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((a^4*x^4 + 2*a^2*x^2 + (a^2*
x^2 + 1)*(a^2*x^2 - 1) + (2*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^4*x^4
 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1)
+ 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\mathrm{arsinh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**2,x)
```

```
[Out] Integral(asinh(a*x)**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-2), x)
```

3.58 $\int \frac{1}{x \sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]^2), x]
```

Rubi [A] time = 0.0131018, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^2} dx = \int \frac{1}{x \sinh^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.72891, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]^2), x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\operatorname{Arcsinh}\left(ax\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^2,x)
```

```
[Out] int(1/x/arcsinh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{3}x^{3}+ax+\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(a^{3}x^{3}+\sqrt{a^{2}x^{2}+1}a^{2}x^{2}+ax\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}-\int\frac{2\left(a^{2}x^{2}+1\right)ax+\left(2\,a^{2}x^{2}+1\right)\sqrt{a^{2}x^{2}+1}}{\left(a^{5}x^{6}+\left(a^{2}x^{2}+1\right)a^{3}x^{4}+2\,a^{3}x^{4}+ax^{2}+2\left(a^{4}x^{5}+a^{2}x^{3}\right)\sqrt{a^{2}x^{2}+1}\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^3 + sqrt(a^2*x^2 + 1)*a^2*x^
2 + a*x)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((2*(a^2*x^2 + 1)*a*x + (
2*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^5*x^6 + (a^2*x^2 + 1)*a^3*x^4 + 2*a^3
*x^4 + a*x^2 + 2*(a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*
x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{x\operatorname{arsinh}\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{asinh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(x*asinh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^2), x)
```

3.59 $\int \frac{1}{x^2 \sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \sinh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcSinh[a*x]^2), x]
```

Rubi [A] time = 0.0136163, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^2} \, dx = \int \frac{1}{x^2 \sinh^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 4.76844, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*ArcSinh[a*x]^2), x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\mathrm{Arcsinh}\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^2,x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^3 + ax + \left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^3x^4 + \sqrt{a^2x^2+1}a^2x^3 + ax^2\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} - \int \frac{a^5x^5 + 2\,a^3x^3 + \left(a^3x^3 + 3\,ax\right)\left(a^2x^2+1\right) + ax + \left(2\,a^4x^4}{\left(a^5x^7 + \left(a^2x^2+1\right)a^3x^5 + 2\,a^3x^5 + ax^3 + 2\left(a^4x^6 + a^2x^4\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^4 + sqrt(a^2*x^2 + 1)*a^2*x^
3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^5*x^5 + 2*a^3*x^3 +
 (a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + a*x + (2*a^4*x^4 + 5*a^2*x^2 + 2)*sqrt(a
^2*x^2 + 1))/((a^5*x^7 + (a^2*x^2 + 1)*a^3*x^5 + 2*a^3*x^5 + a*x^3 + 2*(a^4
*x^6 + a^2*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^2\,\mathrm{arsinh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arcsinh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)^2), x)
```

3.60 $\int \frac{x^4}{\sinh^{-1}(ax)^3} dx$

Optimal. Leaf size=97

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{16a^5}-\frac{27\text{Chi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{25\text{Chi}\left(5\sinh^{-1}(ax)\right)}{32a^5}-\frac{x^4\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{2x^3}{a^2\sinh^{-1}(ax)}-\frac{5x^5}{2\sinh^{-1}(ax)}$$

```
[Out] -(x^4*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - (2*x^3)/(a^2*ArcSinh[a*x])
- (5*x^5)/(2*ArcSinh[a*x]) + CoshIntegral[ArcSinh[a*x]]/(16*a^5) - (27*Cosh
Integral[3*ArcSinh[a*x]])/(32*a^5) + (25*CoshIntegral[5*ArcSinh[a*x]])/(32*
a^5)
```

Rubi [A] time = 0.354339, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5667, 5774, 5669, 5448, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{16a^5}-\frac{27\text{Chi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{25\text{Chi}\left(5\sinh^{-1}(ax)\right)}{32a^5}-\frac{x^4\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{2x^3}{a^2\sinh^{-1}(ax)}-\frac{5x^5}{2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^3,x]
```

```
[Out] -(x^4*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - (2*x^3)/(a^2*ArcSinh[a*x])
- (5*x^5)/(2*ArcSinh[a*x]) + CoshIntegral[ArcSinh[a*x]]/(16*a^5) - (27*Cosh
Integral[3*ArcSinh[a*x]])/(32*a^5) + (25*CoshIntegral[5*ArcSinh[a*x]])/(32*
a^5)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^3} dx = -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{2\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2} dx}{a} + \frac{1}{2}(5a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2} dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{25}{2}\int \frac{x^4}{\sinh^{-1}(ax)} dx + \frac{6\int \frac{x^2}{\sinh^{-1}(ax)} dx}{a^2}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{6\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x} dx, x, \sinh^{-1}(ax)\right)}{a^5} +$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{6\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right) dx, x, \sinh^{-1}(\right.}{a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{25\,\text{Subst}\left(\int \frac{\cosh(5x)}{x} dx, x, \sinh^{-1}(ax)\right)}{32a^5} - \frac{3\,\text{Su}}{}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{16a^5} - \frac{27\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{32a^5} +$$

Mathematica [A] time = 0.140226, size = 102, normalized size = 1.05

$$-\frac{16a^4x^4\sqrt{a^2x^2+1} + 80a^5x^5\sinh^{-1}(ax) + 64a^3x^3\sinh^{-1}(ax) - 2\sinh^{-1}(ax)^2\textbf{Chi}\left(\sinh^{-1}(ax)\right) + 27\sinh^{-1}(ax)^2\textbf{Chi}\left(3\right.}{32a^5\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x]^3,x]
```

```
[Out] -(16*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5*ArcSi
nh[a*x] - 2*ArcSinh[a*x]^2*CoshIntegral[ArcSinh[a*x]] + 27*ArcSinh[a*x]^2*C
oshIntegral[3*ArcSinh[a*x]] - 25*ArcSinh[a*x]^2*CoshIntegral[5*ArcSinh[a*x]
])/(32*a^5*ArcSinh[a*x]^2)
```

Maple [A] time = 0.039, size = 120, normalized size = 1.2

$$\frac{1}{a^5}\left(-\frac{1}{16\,(\text{Arcsinh}\,(ax))^2}\sqrt{a^2x^2+1} - \frac{ax}{16\,\text{Arcsinh}\,(ax)} + \frac{\text{Chi}\,(\text{Arcsinh}\,(ax))}{16} + \frac{3\,\cosh\,(3\,\text{Arcsinh}\,(ax))}{32\,(\text{Arcsinh}\,(ax))^2} + \frac{9\,\sinh\,(3\,\text{A}}{32\,\text{Arcs}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^5*(-1/16/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/16*a*x/arcsinh(a*x)+1/16*Ch
i(arcsinh(a*x))+3/32/arcsinh(a*x)^2*cosh(3*arcsinh(a*x))+9/32/arcsinh(a*x)*
sinh(3*arcsinh(a*x))-27/32*Chi(3*arcsinh(a*x))-1/32/arcsinh(a*x)^2*cosh(5*a
rcsinh(a*x))-5/32/arcsinh(a*x)*sinh(5*arcsinh(a*x))+25/32*Chi(5*arcsinh(a*x
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^11 + 3*a^6*x^9 + 3*a^4*x^7 + a^2*x^5 + (a^5*x^8 + a^3*x^6)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^9 + 5*a^4*x^7 + 2*a^2*x^5)*(a^2*x^2 + 1) + (5*a^
8*x^11 + 15*a^6*x^9 + 15*a^4*x^7 + 5*a^2*x^5 + (5*a^5*x^8 + 8*a^3*x^6 + 3*a
*x^4)*(a^2*x^2 + 1)^(3/2) + (15*a^6*x^9 + 31*a^4*x^7 + 20*a^2*x^5 + 4*x^3)*
(a^2*x^2 + 1) + (15*a^7*x^10 + 38*a^5*x^8 + 32*a^3*x^6 + 9*a*x^4)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^10 + 7*a^5*x^8 + 5*a^3*x^
6 + a*x^4)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a
^5*x^3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5
 + 2*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) +
integrate(1/2*(25*a^10*x^12 + 100*a^8*x^10 + 150*a^6*x^8 + 100*a^4*x^6 + 25
*a^2*x^4 + (25*a^6*x^8 + 24*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^2 + (100*a^7
*x^9 + 172*a^5*x^7 + 87*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(50*a^8
*x^10 + 124*a^6*x^8 + 105*a^4*x^6 + 35*a^2*x^4 + 4*x^2)*(a^2*x^2 + 1) + (10
0*a^9*x^11 + 324*a^7*x^9 + 381*a^5*x^7 + 193*a^3*x^5 + 36*a*x^3)*sqrt(a^2*x
^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a
^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4
 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*
x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\operatorname{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**3,x)
```

```
[Out] Integral(x**4/asinh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^3, x)
```

3.61 $\int \frac{x^3}{\sinh^{-1}(ax)^3} dx$

Optimal. Leaf size=82

$$-\frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{3x^2}{2a^2\sinh^{-1}(ax)}-\frac{2x^4}{\sinh^{-1}(ax)}$$

```
[Out] -(x^3*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - (3*x^2)/(2*a^2*ArcSinh[a*x]
) - (2*x^4)/ArcSinh[a*x] - SinhIntegral[2*ArcSinh[a*x]]/(2*a^4) + SinhInteg
ral[4*ArcSinh[a*x]]/a^4
```

Rubi [A] time = 0.304009, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5667, 5774, 5669, 5448, 3298, 12}

$$-\frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{3x^2}{2a^2\sinh^{-1}(ax)}-\frac{2x^4}{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^3,x]
```

```
[Out] -(x^3*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - (3*x^2)/(2*a^2*ArcSinh[a*x]
) - (2*x^4)/ArcSinh[a*x] - SinhIntegral[2*ArcSinh[a*x]]/(2*a^4) + SinhInteg
ral[4*ArcSinh[a*x]]/a^4
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
```

```
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sinh^{-1}(ax)^3}\,dx &= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{2a} + (2a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + 8\int \frac{x^3}{\sinh^{-1}(ax)}\,dx + \frac{3\int \frac{x}{\sinh^{-1}(ax)}\,dx}{a^2} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} + \frac{8\,\mathrm{S}}{} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} + \frac{8\,\mathrm{Subst}}{} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} + \frac{3\,\mathrm{Subst}\,(}{} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} - \frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^4} + \frac{\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{a^4}
\end{aligned}
$$

Mathematica [A] time = 0.203097, size = 69, normalized size = 0.84

$$
-\frac{\frac{a^2x^2\left(ax\sqrt{a^2x^2+1}+\left(4a^2x^2+3\right)\sinh^{-1}(ax)\right)}{\sinh^{-1}(ax)^2} + \mathrm{Shi}\left(2\sinh^{-1}(ax)\right) - 2\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{2a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x]^3,x]
```

```
[Out] -((a^2*x^2*(a*x*Sqrt[1 + a^2*x^2] + (3 + 4*a^2*x^2)*ArcSinh[a*x]))/ArcSinh[
a*x]^2 + SinhIntegral[2*ArcSinh[a*x]] - 2*SinhIntegral[4*ArcSinh[a*x]])/(2*
a^4)
```

Maple [A] time = 0.03, size = 82, normalized size = 1.

$$
\frac{1}{a^4}\left(\frac{\sinh(2\,\mathrm{Arcsinh}\,(ax))}{8\,(\mathrm{Arcsinh}\,(ax))^2} + \frac{\cosh(2\,\mathrm{Arcsinh}\,(ax))}{4\,\mathrm{Arcsinh}\,(ax)} - \frac{\mathrm{Shi}\,(2\,\mathrm{Arcsinh}\,(ax))}{2} - \frac{\sinh(4\,\mathrm{Arcsinh}\,(ax))}{16\,(\mathrm{Arcsinh}\,(ax))^2} - \frac{\cosh(4\,\mathrm{Arcsinh}}{4\,\mathrm{Arcsinh}\,(ax}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/8/arcsinh(a*x)^2*sinh(2*arcsinh(a*x))+1/4/arcsinh(a*x)*cosh(2*arcs
inh(a*x))-1/2*Shi(2*arcsinh(a*x))-1/16/arcsinh(a*x)^2*sinh(4*arcsinh(a*x))-
1/4/arcsinh(a*x)*cosh(4*arcsinh(a*x))+Shi(4*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^10 + 3*a^6*x^8 + 3*a^4*x^6 + a^2*x^4 + (a^5*x^7 + a^3*x^5)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^8 + 5*a^4*x^6 + 2*a^2*x^4)*(a^2*x^2 + 1) + (4*a^
8*x^10 + 12*a^6*x^8 + 12*a^4*x^6 + 4*a^2*x^4 + 2*(2*a^5*x^7 + 3*a^3*x^5 + a
*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(4*a^6*x^8 + 8*a^4*x^6 + 5*a^2*x^4 + x^2)*(a^
2*x^2 + 1) + (12*a^7*x^9 + 30*a^5*x^7 + 25*a^3*x^5 + 7*a*x^3)*sqrt(a^2*x^2
+ 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^9 + 7*a^5*x^7 + 5*a^3*x^5 + a
*x^3)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^
3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*
a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integ
rate(1/2*(16*a^10*x^11 + 64*a^8*x^9 + 96*a^6*x^7 + 64*a^4*x^5 + 16*a^2*x^3
+ 4*(4*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^2 + (64*a^7*x^8 + 100*a^5*x^6 + 4
2*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^(3/2) + 6*(16*a^8*x^9 + 38*a^6*x^7 + 30*
a^4*x^5 + 9*a^2*x^3 + x)*(a^2*x^2 + 1) + (64*a^9*x^10 + 204*a^7*x^8 + 234*a
^5*x^6 + 115*a^3*x^4 + 21*a*x^2)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6
+ (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(
a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2
+ 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x +
sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\operatorname{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**3,x)
```

```
[Out] Integral(x**3/asinh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x)^3, x)
```

3.62 $\int \frac{x^2}{\sinh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{x}{a^2\sinh^{-1}(ax)}-\frac{3x^3}{2\sinh^{-1}(ax)}$$

```
[Out] -(x^2*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - x/(a^2*ArcSinh[a*x]) - (3*x
^3)/(2*ArcSinh[a*x]) - CoshIntegral[ArcSinh[a*x]]/(8*a^3) + (9*CoshIntegral
[3*ArcSinh[a*x]])/(8*a^3)
```

Rubi [A] time = 0.252573, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5667, 5774, 5669, 5448, 3301, 5657}

$$-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{x}{a^2\sinh^{-1}(ax)}-\frac{3x^3}{2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^3,x]
```

```
[Out] -(x^2*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - x/(a^2*ArcSinh[a*x]) - (3*x
^3)/(2*ArcSinh[a*x]) - CoshIntegral[ArcSinh[a*x]]/(8*a^3) + (9*CoshIntegral
[3*ArcSinh[a*x]])/(8*a^3)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
```

```
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^3}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(3a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{9}{2}\int \frac{x^2}{\sinh^{-1}(ax)}\,dx + \frac{\int \frac{1}{\sinh^{-1}(ax)}\,dx}{a^2}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4x} + \right.\right.}{2}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{a^3} - \frac{9\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \right.}{8a^3}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} - \frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{8a^3} + \frac{9\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}$$

Mathematica [A] time = 0.136761, size = 64, normalized size = 0.79

$$-\frac{\frac{4ax\left(ax\sqrt{a^2x^2+1}+\left(3a^2x^2+2\right)\sinh^{-1}(ax)\right)}{\sinh^{-1}(ax)^2} + \textbf{Chi}\left(\sinh^{-1}(ax)\right) - 9\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x]^3,x]
```

```
[Out] -((4*a*x*(a*x*Sqrt[1 + a^2*x^2] + (2 + 3*a^2*x^2)*ArcSinh[a*x]))/ArcSinh[a*
x]^2 + CoshIntegral[ArcSinh[a*x]] - 9*CoshIntegral[3*ArcSinh[a*x]])/(8*a^3)
```

Maple [A] time = 0.029, size = 81, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{1}{8\left(\text{Arcsinh}(ax)\right)^2}\sqrt{a^2x^2+1} + \frac{ax}{8\,\text{Arcsinh}(ax)} - \frac{\text{Chi}(\text{Arcsinh}(ax))}{8} - \frac{\cosh(3\,\text{Arcsinh}(ax))}{8\left(\text{Arcsinh}(ax)\right)^2} - \frac{3\sinh(3\,\text{Arcsin}}{8\,\text{Arcsinh}(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/8/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+1/8*a*x/arcsinh(a*x)-1/8*Chi(ar
csinh(a*x))-1/8/arcsinh(a*x)^2*cosh(3*arcsinh(a*x))-3/8/arcsinh(a*x)*sinh(3
*arcsinh(a*x))+9/8*Chi(3*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^9 + 3*a^6*x^7 + 3*a^4*x^5 + a^2*x^3 + (a^5*x^6 + a^3*x^4)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^7 + 5*a^4*x^5 + 2*a^2*x^3)*(a^2*x^2 + 1) + (3*a^8
*x^9 + 9*a^6*x^7 + 9*a^4*x^5 + 3*a^2*x^3 + (3*a^5*x^6 + 4*a^3*x^4 + a*x^2)*
(a^2*x^2 + 1)^(3/2) + (9*a^6*x^7 + 17*a^4*x^5 + 10*a^2*x^3 + 2*x)*(a^2*x^2
+ 1) + (9*a^7*x^8 + 22*a^5*x^6 + 18*a^3*x^4 + 5*a*x^2)*sqrt(a^2*x^2 + 1))*l
og(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^8 + 7*a^5*x^6 + 5*a^3*x^4 + a*x^2)*s
qrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a
^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3
 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/
2*(9*a^10*x^10 + 36*a^8*x^8 + 54*a^6*x^6 + 36*a^4*x^4 + 9*a^2*x^2 + (9*a^6*
x^6 + 4*a^4*x^4 - a^2*x^2)*(a^2*x^2 + 1)^2 + (36*a^7*x^7 + 48*a^5*x^5 + 13*
a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^(3/2) + (54*a^8*x^8 + 120*a^6*x^6 + 83*a^4*x
^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1) + (36*a^9*x^9 + 112*a^7*x^7 + 123*a^5*x^
5 + 57*a^3*x^3 + 10*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x
^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2
+ 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9
*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2
*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\operatorname{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**3,x)
```

```
[Out] Integral(x**2/asinh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^3, x)
```

3.63 $\int \frac{x}{\sinh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=63

$$\frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}$$

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - 1/(2*a^2*ArcSinh[a*x]) - x^2/
ArcSinh[a*x] + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Rubi [A] time = 0.170711, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5667, 5774, 5669, 5448, 12, 3298, 5675}

$$\frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^3,x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - 1/(2*a^2*ArcSinh[a*x]) - x^2/
ArcSinh[a*x] + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(ax)^3}\,dx = -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{2a} + a\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx$$
$$= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{1}{2a^2\sinh^{-1}(ax)} - \frac{x^2}{\sinh^{-1}(ax)} + 2\int \frac{x}{\sinh^{-1}(ax)}\,dx$$
$$= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{1}{2a^2\sinh^{-1}(ax)} - \frac{x^2}{\sinh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{1}{2a^2\sinh^{-1}(ax)} - \frac{x^2}{\sinh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{1}{2a^2\sinh^{-1}(ax)} - \frac{x^2}{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{1}{2a^2\sinh^{-1}(ax)} - \frac{x^2}{\sinh^{-1}(ax)} + \frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0492223, size = 62, normalized size = 0.98

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2} - \frac{x\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2} + \frac{-2a^2x^2-1}{2a^2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x]^3,x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) + (-1 - 2*a^2*x^2)/(2*a^2*ArcSi
nh[a*x]) + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Maple [A] time = 0.03, size = 43, normalized size = 0.7

$$\frac{1}{a^2}\left(-\frac{\sinh(2\,\text{Arcsinh}(ax))}{4\,(\text{Arcsinh}(ax))^2} - \frac{\cosh(2\,\text{Arcsinh}(ax))}{2\,\text{Arcsinh}(ax)} + \text{Shi}(2\,\text{Arcsinh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^2*(-1/4/arcsinh(a*x)^2*sinh(2*arcsinh(a*x))-1/2/arcsinh(a*x)*cosh(2*arc
sinh(a*x))+Shi(2*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) + (2*a^8
*x^8 + 6*a^6*x^6 + 6*a^4*x^4 + 2*a^2*x^2 + 2*(a^5*x^5 + a^3*x^3)*(a^2*x^2 +
 1)^(3/2) + (6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (6*a^7
*x^7 + 14*a^5*x^5 + 11*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a
^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))
/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a^4*x^2 + 3*(a^6*x
^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3 + a^3*x)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*a^9*x^9 + 16
*a^7*x^7 + 4*(a^2*x^2 + 1)^2*a^5*x^5 + 24*a^5*x^5 + 16*a^3*x^3 + (16*a^6*x^
6 + 16*a^4*x^4 - 3)*(a^2*x^2 + 1)^(3/2) + 24*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3
)*(a^2*x^2 + 1) + 4*a*x + (16*a^8*x^8 + 48*a^6*x^6 + 48*a^4*x^4 + 19*a^2*x^
2 + 3)*sqrt(a^2*x^2 + 1))/((a^9*x^8 + 4*a^7*x^6 + (a^2*x^2 + 1)^2*a^5*x^4 +
 6*a^5*x^4 + 4*a^3*x^2 + 4*(a^6*x^5 + a^4*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^7
*x^6 + 2*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 4*(a^8*x^7 + 3*a^6*x^5 + 3*a^4*
x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\operatorname{arsinh}\left(ax\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**3,x)
```

```
[Out] Integral(x/asinh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^3, x)
```

3.64 $\int \frac{1}{\sinh^{-1}(ax)^3} dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}+\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{2a}-\frac{x}{2\sinh^{-1}(ax)}$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a*ArcSinh[a*x]^2) - x/(2*ArcSinh[a*x]) + CoshIntegral
[ArcSinh[a*x]]/(2*a)
```

Rubi [A] time = 0.0826796, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5655, 5774, 5657, 3301}

$$-\frac{\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}+\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{2a}-\frac{x}{2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-3),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a*ArcSinh[a*x]^2) - x/(2*ArcSinh[a*x]) + CoshIntegral
[ArcSinh[a*x]]/(2*a)
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{-1}(ax)^3}\,dx &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{x}{2\sinh^{-1}(ax)}+\frac{1}{2}\int \frac{1}{\sinh^{-1}(ax)}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{x}{2\sinh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{2a}\\ &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{x}{2\sinh^{-1}(ax)}+\frac{\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0184735, size = 47, normalized size = 0.94

$$-\frac{\sqrt{a^2x^2+1}+\sinh^{-1}(ax)^2\left(-\textbf{Chi}\left(\sinh^{-1}(ax)\right)\right)+ax\sinh^{-1}(ax)}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-3),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] - ArcSinh[a*x]^2*CoshIntegral[ArcSin
h[a*x]])/(2*a*ArcSinh[a*x]^2)
```

Maple [A] time = 0.022, size = 42, normalized size = 0.8

$$\frac{1}{a}\left(-\frac{1}{2\,(\text{Arcsinh}\,(ax))^2}\sqrt{a^2x^2+1}-\frac{ax}{2\,\text{Arcsinh}\,(ax)}+\frac{\text{Chi}\,(\text{Arcsinh}\,(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^3,x)
```

```
[Out] 1/a*(-1/2/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/2*a*x/arcsinh(a*x)+1/2*Chi(arc
sinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 + 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^(
3/2) + (3*a^5*x^5 + 5*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + a*x + (a^7*x^7 + 3*a
^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - 1)*(a^2*x^2 + 1)^(3/2) + 3*(a^5*x^5 + a^3*x
^3)*(a^2*x^2 + 1) + a*x + (3*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + 1)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^6*x^6 + 7*a^4*x^4 + 5*a^2*x^2
 + 1)*sqrt(a^2*x^2 + 1))/((a^7*x^6 + 3*a^5*x^4 + (a^2*x^2 + 1)^(3/2)*a^4*x^
3 + 3*a^3*x^2 + 3*(a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 3*(a^6*x^5 + 2*a^4*x^
3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^2) + integra
te(1/2*(a^8*x^8 + 4*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + (a^4*x^4 + 3)*(a^2*x^
2 + 1)^2 + (4*a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^6*x
^6 + 4*a^4*x^4 + a^2*x^2 - 1)*(a^2*x^2 + 1) + (4*a^7*x^7 + 12*a^5*x^5 + 9*a
^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^8*x^8 + 4*a^6*x^6 + (a^2*x^2 + 1)^
2*a^4*x^4 + 6*a^4*x^4 + 4*a^2*x^2 + 4*(a^5*x^5 + a^3*x^3)*(a^2*x^2 + 1)^(3/
2) + 6*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 4*(a^7*x^7 + 3*a^5*x
^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**3,x)
```

```
[Out] Integral(asinh(a*x)**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-3), x)
```

3.65 $\int \frac{1}{x \sinh^{-1}(ax)^3} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]^3), x]
```

Rubi [A] time = 0.0131602, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^3} dx = \int \frac{1}{x \sinh^{-1}(ax)^3} dx$$

Mathematica [A] time = 0.540267, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]^3), x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\mathrm{Arcsinh}\left(ax\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^3,x)
```

```
[Out] int(1/x/arcsinh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{8}x^{8}+3\,a^{6}x^{6}+3\,a^{4}x^{4}+a^{2}x^{2}+(a^{5}x^{5}+a^{3}x^{3})(a^{2}x^{2}+1)^{\frac{3}{2}}+(3\,a^{6}x^{6}+5\,a^{4}x^{4}+2\,a^{2}x^{2})(a^{2}x^{2}+1)-(2\,(a^{3}x^{3}+ax)(a^{2}x^{2}}{2\,(a^{8}x^{8}+3\,a^{6}x^{6}+(a^{2}x^{2}+1)^{\frac{3}{2}}a^{5}x^{5}+3\,a^{4}x^{4}+a^{2}x^{2}+3\,(a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (2*(a^
3*x^3 + a*x)*(a^2*x^2 + 1)^(3/2) + (4*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1
) + (2*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^8
*x^8 + 3*a^6*x^6 + (a^2*x^2 + 1)^(3/2)*a^5*x^5 + 3*a^4*x^4 + a^2*x^2 + 3*(a
^6*x^6 + a^4*x^4)*(a^2*x^2 + 1) + 3*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*(a^4*x^4 + 2
*a^2*x^2)*(a^2*x^2 + 1)^2 + (12*a^5*x^5 + 22*a^3*x^3 + 7*a*x)*(a^2*x^2 + 1)
^(3/2) + 2*(6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (4*a^7*
x^7 + 6*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^11 + 4*a^8*x
^9 + (a^2*x^2 + 1)^2*a^6*x^7 + 6*a^6*x^7 + 4*a^4*x^5 + a^2*x^3 + 4*(a^7*x^8
 + a^5*x^6)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^9 + 2*a^6*x^7 + a^4*x^5)*(a^2*x^
2 + 1) + 4*(a^9*x^10 + 3*a^7*x^8 + 3*a^5*x^6 + a^3*x^4)*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\,\text{arsinh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{asinh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**3,x)
```

```
[Out] Integral(1/(x*asinh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{arsinh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^3), x)
```

3.66 $\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \sinh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcSinh[a*x]^3), x]
```

Rubi [A] time = 0.0137973, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx = \int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 4.9924, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*ArcSinh[a*x]^3), x]
```

Maple [A] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (\operatorname{Arcsinh}(ax))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^3,x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (a^8*x
^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x
^2 + 1)^(3/2) + (3*a^6*x^6 + 11*a^4*x^4 + 10*a^2*x^2 + 2)*(a^2*x^2 + 1) + (
3*a^7*x^7 + 10*a^5*x^5 + 10*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + s
qrt(a^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2
+ 1))/((a^8*x^9 + 3*a^6*x^7 + (a^2*x^2 + 1)^(3/2)*a^5*x^6 + 3*a^4*x^5 + a^2
*x^3 + 3*(a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1) + 3*(a^7*x^8 + 2*a^5*x^6 + a^3*x
^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(a^1
0*x^10 + 4*a^8*x^8 + 6*a^6*x^6 + 4*a^4*x^4 + a^2*x^2 + (a^6*x^6 + 12*a^4*x^
4 + 15*a^2*x^2)*(a^2*x^2 + 1)^2 + (4*a^7*x^7 + 40*a^5*x^5 + 57*a^3*x^3 + 18
*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^8*x^8 + 16*a^6*x^6 + 25*a^4*x^4 + 13*a^2
*x^2 + 2)*(a^2*x^2 + 1) + (4*a^9*x^9 + 24*a^7*x^7 + 39*a^5*x^5 + 25*a^3*x^3
 + 6*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^12 + 4*a^8*x^10 + (a^2*x^2 + 1)^2*a^6
*x^8 + 6*a^6*x^8 + 4*a^4*x^6 + a^2*x^4 + 4*(a^7*x^9 + a^5*x^7)*(a^2*x^2 + 1
)^(3/2) + 6*(a^8*x^10 + 2*a^6*x^8 + a^4*x^6)*(a^2*x^2 + 1) + 4*(a^9*x^11 +
3*a^7*x^9 + 3*a^5*x^7 + a^3*x^5)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2\,\text{arsinh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arcsinh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,\text{asinh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**3,x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,\text{arsinh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)^3), x)
```

3.67 $\int \frac{x^4}{\sinh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=155

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{48a^5}-\frac{27\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{125\mathrm{Shi}\left(5\sinh^{-1}(ax)\right)}{96a^5}-\frac{25x^4\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{x^4\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-1}(ax}$$

```
[Out] -(x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - (2*x^3)/(3*a^2*ArcSinh[a*x]
^2) - (5*x^5)/(6*ArcSinh[a*x]^2) - (2*x^2*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a
*x]) - (25*x^4*Sqrt[1 + a^2*x^2])/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh
[a*x]]/(48*a^5) - (27*SinhIntegral[3*ArcSinh[a*x]])/(32*a^5) + (125*SinhInt
egral[5*ArcSinh[a*x]])/(96*a^5)
```

Rubi [A] time = 0.319825, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5667, 5774, 5665, 3298}

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{48a^5}-\frac{27\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{125\mathrm{Shi}\left(5\sinh^{-1}(ax)\right)}{96a^5}-\frac{25x^4\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{x^4\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-1}(ax}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^4,x]
```

```
[Out] -(x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - (2*x^3)/(3*a^2*ArcSinh[a*x]
^2) - (5*x^5)/(6*ArcSinh[a*x]^2) - (2*x^2*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a
*x]) - (25*x^4*Sqrt[1 + a^2*x^2])/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh
[a*x]]/(48*a^5) - (27*SinhIntegral[3*ArcSinh[a*x]])/(32*a^5) + (125*SinhInt
egral[5*ArcSinh[a*x]])/(96*a^5)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
 - 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sinh^{-1}(ax)^4}\,dx &= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}+\frac{4\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{3a}+\frac{1}{3}(5a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx\\
&= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-1}(ax)^2}-\frac{5x^5}{6\sinh^{-1}(ax)^2}+\frac{25}{6}\int \frac{x^4}{\sinh^{-1}(ax)^2}\,dx+\frac{2\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-1}(ax)^2}-\frac{5x^5}{6\sinh^{-1}(ax)^2}-\frac{2x^2\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)}-\frac{25x^4\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)}+\frac{2\,\mathrm{Subst}\big(\\
&= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-1}(ax)^2}-\frac{5x^5}{6\sinh^{-1}(ax)^2}-\frac{2x^2\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)}-\frac{25x^4\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)}-\frac{\mathrm{Subst}\big(\int\\
&= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-1}(ax)^2}-\frac{5x^5}{6\sinh^{-1}(ax)^2}-\frac{2x^2\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)}-\frac{25x^4\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)}+\frac{\mathrm{Shi}\big(\sinh}{48a}\end{aligned}$$

Mathematica [A] time = 0.328406, size = 156, normalized size = 1.01

$$-\frac{32a^4x^4\sqrt{a^2x^2+1}+80a^5x^5\sinh^{-1}(ax)+400a^4x^4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2+64a^3x^3\sinh^{-1}(ax)+192a^2x^2\sqrt{a^2x^2+1}\sinh}{96a^5\sinh^{-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x]^4,x]
```

```
[Out] -(32*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5*ArcSi
nh[a*x] + 192*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 400*a^4*x^4*Sqrt[1
 + a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x]] +
81*ArcSinh[a*x]^3*SinhIntegral[3*ArcSinh[a*x]] - 125*ArcSinh[a*x]^3*SinhInt
egral[5*ArcSinh[a*x]])/(96*a^5*ArcSinh[a*x]^3)
```

Maple [A] time = 0.046, size = 169, normalized size = 1.1

$$\frac{1}{a^5}\left(-\frac{1}{24\,(\mathrm{Arcsinh}\,(ax))^3}\sqrt{a^2x^2+1}-\frac{ax}{48\,(\mathrm{Arcsinh}\,(ax))^2}-\frac{1}{48\,\mathrm{Arcsinh}\,(ax)}\sqrt{a^2x^2+1}+\frac{\mathrm{Shi}\,(\mathrm{Arcsinh}\,(ax))}{48}+\frac{\cosh\,(}{16\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^5*(-1/24/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-1/48*a*x/arcsinh(a*x)^2-1/48/
arcsinh(a*x)*(a^2*x^2+1)^(1/2)+1/48*Shi(arcsinh(a*x))+1/16/arcsinh(a*x)^3*c
osh(3*arcsinh(a*x))+3/32/arcsinh(a*x)^2*sinh(3*arcsinh(a*x))+9/32/arcsinh(a
*x)*cosh(3*arcsinh(a*x))-27/32*Shi(3*arcsinh(a*x))-1/48/arcsinh(a*x)^3*cosh
(5*arcsinh(a*x))-5/96/arcsinh(a*x)^2*sinh(5*arcsinh(a*x))-25/96/arcsinh(a*x
)*cosh(5*arcsinh(a*x))+125/96*Shi(5*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^15 + 10*a^11*x^13 + 20*a^9*x^11 + 20*a^7*x^9 + 10*a^5*x^7 +
2*a^3*x^5 + 2*(a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^11 + 9*
a^7*x^9 + 4*a^5*x^7)*(a^2*x^2 + 1)^2 + (25*a^13*x^15 + 125*a^11*x^13 + 250*
a^9*x^11 + 250*a^7*x^9 + 125*a^5*x^7 + 25*a^3*x^5 + (25*a^8*x^10 + 49*a^6*x
^8 + 27*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^(5/2) + (125*a^9*x^11 + 321*a^7*
```

```
x^9 + 286*a^5*x^7 + 102*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^2 + (250*a^10*x^1
2 + 794*a^8*x^10 + 946*a^6*x^8 + 519*a^4*x^6 + 129*a^2*x^4 + 12*x^2)*(a^2*x
^2 + 1)^(3/2) + 2*(125*a^11*x^13 + 473*a^9*x^11 + 696*a^7*x^9 + 497*a^5*x^7
 + 173*a^3*x^5 + 24*a*x^3)*(a^2*x^2 + 1) + (125*a^12*x^14 + 549*a^10*x^12 +
 955*a^8*x^10 + 824*a^6*x^8 + 354*a^4*x^6 + 61*a^2*x^4)*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^12 + 13*a^8*x^10 + 11*a^6*x^8
+ 3*a^4*x^6)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^13 + 17*a^9*x^11 + 21*a^7*x^
9 + 11*a^5*x^7 + 2*a^3*x^5)*(a^2*x^2 + 1) + (5*a^13*x^15 + 25*a^11*x^13 + 5
0*a^9*x^11 + 50*a^7*x^9 + 25*a^5*x^7 + 5*a^3*x^5 + (5*a^8*x^10 + 8*a^6*x^8
+ 3*a^4*x^6)*(a^2*x^2 + 1)^(5/2) + (25*a^9*x^11 + 57*a^7*x^9 + 42*a^5*x^7 +
 10*a^3*x^5)*(a^2*x^2 + 1)^2 + (50*a^10*x^12 + 148*a^8*x^10 + 158*a^6*x^8 +
 71*a^4*x^6 + 11*a^2*x^4)*(a^2*x^2 + 1)^(3/2) + 2*(25*a^11*x^13 + 91*a^9*x^
11 + 126*a^7*x^9 + 81*a^5*x^7 + 23*a^3*x^5 + 2*a*x^3)*(a^2*x^2 + 1) + (25*a
^12*x^14 + 108*a^10*x^12 + 183*a^8*x^10 + 151*a^6*x^8 + 60*a^4*x^6 + 9*a^2*
x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^14 + 21*
a^10*x^12 + 34*a^8*x^10 + 26*a^6*x^8 + 9*a^4*x^6 + a^2*x^4)*sqrt(a^2*x^2 +
1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x^2 + 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 + 1
0*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 10*(a
^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^8 + 3*a^9*x
^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 5*(a^12*x^9 + 4*a^10*x^7 + 6*a^8*
x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3)
 + integrate(1/6*(125*a^15*x^16 + 750*a^13*x^14 + 1875*a^11*x^12 + 2500*a^9
*x^10 + 1875*a^7*x^8 + 750*a^5*x^6 + 125*a^3*x^4 + (125*a^9*x^10 + 147*a^7*
x^8 + 27*a^5*x^6 - 3*a^3*x^4)*(a^2*x^2 + 1)^3 + (750*a^10*x^11 + 1485*a^8*x
^9 + 901*a^6*x^7 + 147*a^4*x^5 - 12*a^2*x^3)*(a^2*x^2 + 1)^(5/2) + (1875*a^
11*x^12 + 5220*a^9*x^10 + 5209*a^7*x^8 + 2185*a^5*x^6 + 321*a^3*x^4)*(a^2*x
^2 + 1)^2 + (2500*a^12*x^13 + 8970*a^10*x^11 + 12366*a^8*x^9 + 8143*a^6*x^7
 + 2583*a^4*x^5 + 360*a^2*x^3 + 24*x)*(a^2*x^2 + 1)^(3/2) + (1875*a^13*x^14
 + 8235*a^11*x^12 + 14449*a^9*x^10 + 12834*a^7*x^8 + 6030*a^5*x^6 + 1429*a^
3*x^4 + 144*a*x^2)*(a^2*x^2 + 1) + (750*a^14*x^15 + 3897*a^12*x^13 + 8293*a
^10*x^11 + 9226*a^8*x^9 + 5655*a^6*x^7 + 1819*a^4*x^5 + 244*a^2*x^3)*sqrt(a
^2*x^2 + 1))/((a^15*x^12 + 6*a^13*x^10 + 15*a^11*x^8 + (a^2*x^2 + 1)^3*a^9*
x^6 + 20*a^9*x^6 + 15*a^7*x^4 + 6*a^5*x^2 + 6*(a^10*x^7 + a^8*x^5)*(a^2*x^2
 + 1)^(5/2) + 15*(a^11*x^8 + 2*a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 2
0*(a^12*x^9 + 3*a^10*x^7 + 3*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a
^13*x^10 + 4*a^11*x^8 + 6*a^9*x^6 + 4*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 6*
(a^14*x^11 + 5*a^12*x^9 + 10*a^10*x^7 + 10*a^8*x^5 + 5*a^6*x^3 + a^4*x)*sqr
t(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**4,x)
```

```
[Out] Integral(x**4/asinh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^4, x)
```

3.68 $\int \frac{x^3}{\sinh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=141

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^4}+\frac{4\text{Chi}\left(4\sinh^{-1}(ax)\right)}{3a^4}-\frac{8x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{x^2}{2a^2\sinh^{-1}(ax)^2}-\frac{x\sqrt{a^2x^2+1}}{a^3\sinh^{-1}(ax)}-$$

```
[Out] -(x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - x^2/(2*a^2*ArcSinh[a*x]^2)
- (2*x^4)/(3*ArcSinh[a*x]^2) - (x*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a*x]) - (
8*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) - CoshIntegral[2*ArcSinh[a*x]]/
(3*a^4) + (4*CoshIntegral[4*ArcSinh[a*x]])/(3*a^4)
```

Rubi [A] time = 0.283297, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5667, 5774, 5665, 3301}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^4}+\frac{4\text{Chi}\left(4\sinh^{-1}(ax)\right)}{3a^4}-\frac{8x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{x^2}{2a^2\sinh^{-1}(ax)^2}-\frac{x\sqrt{a^2x^2+1}}{a^3\sinh^{-1}(ax)}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^4,x]
```

```
[Out] -(x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - x^2/(2*a^2*ArcSinh[a*x]^2)
- (2*x^4)/(3*ArcSinh[a*x]^2) - (x*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a*x]) - (
8*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) - CoshIntegral[2*ArcSinh[a*x]]/
(3*a^4) + (4*CoshIntegral[4*ArcSinh[a*x]])/(3*a^4)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sinh^{-1}(ax)^4}\,dx &= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{a} + \frac{1}{3}(4a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} + \frac{8}{3}\int \frac{x^3}{\sinh^{-1}(ax)^2}\,dx + \frac{\int \frac{x}{\sinh^{-1}(ax)^2}\,dx}{a^2} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} + \frac{\mathrm{Subst}\Big(\int \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} + \frac{\mathrm{Chi}\Big(2\,\mathrm{si}}{a} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} - \frac{\mathrm{Chi}\Big(2\,\mathrm{si}}{3}
\end{aligned}$$

Mathematica [A] time = 0.372958, size = 105, normalized size = 0.74

$$-\frac{\frac{ax\left(2a^2x^2\sqrt{a^2x^2+1}+ax\left(4a^2x^2+3\right)\sinh^{-1}(ax)+2\sqrt{a^2x^2+1}\left(8a^2x^2+3\right)\sinh^{-1}(ax)^2\right)}{\sinh^{-1}(ax)^3}+2\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)-8\mathrm{Chi}\left(4\sinh^{-1}(ax)\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x]^4,x]
```

```
[Out] -((a*x*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(3 + 4*a^2*x^2)*ArcSinh[a*x] + 2*
Sqrt[1 + a^2*x^2]*(3 + 8*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + 2*CoshI
ntegral[2*ArcSinh[a*x]] - 8*CoshIntegral[4*ArcSinh[a*x]])/(6*a^4)
```

Maple [A] time = 0.031, size = 114, normalized size = 0.8

$$\frac{1}{a^4}\left(\frac{\sinh(2\operatorname{Arcsinh}(ax))}{12\,(\operatorname{Arcsinh}(ax))^3}+\frac{\cosh(2\operatorname{Arcsinh}(ax))}{12\,(\operatorname{Arcsinh}(ax))^2}+\frac{\sinh(2\operatorname{Arcsinh}(ax))}{6\operatorname{Arcsinh}(ax)}-\frac{\operatorname{Chi}(2\operatorname{Arcsinh}(ax))}{3}-\frac{\sinh(4\operatorname{Arcsinh}}{24\,(\operatorname{Arcsinh}(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^4*(1/12/arcsinh(a*x)^3*sinh(2*arcsinh(a*x))+1/12/arcsinh(a*x)^2*cosh(2*
arcsinh(a*x))+1/6/arcsinh(a*x)*sinh(2*arcsinh(a*x))-1/3*Chi(2*arcsinh(a*x))
-1/24/arcsinh(a*x)^3*sinh(4*arcsinh(a*x))-1/12/arcsinh(a*x)^2*cosh(4*arcsin
h(a*x))-1/3/arcsinh(a*x)*sinh(4*arcsinh(a*x))+4/3*Chi(4*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^14 + 10*a^11*x^12 + 20*a^9*x^10 + 20*a^7*x^8 + 10*a^5*x^6 +
2*a^3*x^4 + 2*(a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^10 + 9*a
^7*x^8 + 4*a^5*x^6)*(a^2*x^2 + 1)^2 + (16*a^13*x^14 + 80*a^11*x^12 + 160*a^
9*x^10 + 160*a^7*x^8 + 80*a^5*x^6 + 16*a^3*x^4 + 4*(4*a^8*x^9 + 7*a^6*x^7 +
 3*a^4*x^5)*(a^2*x^2 + 1)^(5/2) + (80*a^9*x^10 + 192*a^7*x^8 + 154*a^5*x^6
+ 45*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^2 + (160*a^10*x^11 + 488*a^8*x^9 + 55
0*a^6*x^7 + 279*a^4*x^5 + 63*a^2*x^3 + 6*x)*(a^2*x^2 + 1)^(3/2) + (160*a^11
*x^12 + 592*a^9*x^10 + 846*a^7*x^8 + 583*a^5*x^6 + 196*a^3*x^4 + 27*a*x^2)*
(a^2*x^2 + 1) + (80*a^12*x^13 + 348*a^10*x^11 + 598*a^8*x^9 + 509*a^6*x^7 +
 216*a^4*x^5 + 37*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 4*(5*a^10*x^11 + 13*a^8*x^9 + 11*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^(3/
```

```
2) + 4*(5*a^11*x^12 + 17*a^9*x^10 + 21*a^7*x^8 + 11*a^5*x^6 + 2*a^3*x^4)*(a
^2*x^2 + 1) + (4*a^13*x^14 + 20*a^11*x^12 + 40*a^9*x^10 + 40*a^7*x^8 + 20*a
^5*x^6 + 4*a^3*x^4 + 2*(2*a^8*x^9 + 3*a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1)^(5/2
) + (20*a^9*x^10 + 44*a^7*x^8 + 31*a^5*x^6 + 7*a^3*x^4)*(a^2*x^2 + 1)^2 + (
40*a^10*x^11 + 116*a^8*x^9 + 121*a^6*x^7 + 53*a^4*x^5 + 8*a^2*x^3)*(a^2*x^2
 + 1)^(3/2) + (40*a^11*x^12 + 144*a^9*x^10 + 197*a^7*x^8 + 125*a^5*x^6 + 35
*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1) + (20*a^12*x^13 + 86*a^10*x^11 + 145*a^8*
x^9 + 119*a^6*x^7 + 47*a^4*x^5 + 7*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sq
rt(a^2*x^2 + 1)) + 2*(5*a^12*x^13 + 21*a^10*x^11 + 34*a^8*x^9 + 26*a^6*x^7
+ 9*a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x
^2 + 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 + 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 +
a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 10*(a^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x
^2 + 1)^(3/2) + 10*(a^11*x^8 + 3*a^9*x^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 +
1) + 5*(a^12*x^9 + 4*a^10*x^7 + 6*a^8*x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate(1/6*(64*a^15*x^15 + 384*
a^13*x^13 + 960*a^11*x^11 + 1280*a^9*x^9 + 960*a^7*x^7 + 384*a^5*x^5 + 64*a
^3*x^3 + 8*(8*a^9*x^9 + 7*a^7*x^7)*(a^2*x^2 + 1)^3 + (384*a^10*x^10 + 664*a
^8*x^8 + 308*a^6*x^6 + 12*a^4*x^4 - 9*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + 2*(480
*a^11*x^11 + 1240*a^9*x^9 + 1096*a^7*x^7 + 360*a^5*x^5 + 15*a^3*x^3 - 9*a*x
)*(a^2*x^2 + 1)^2 + 2*(640*a^12*x^12 + 2200*a^10*x^10 + 2844*a^8*x^8 + 1684
*a^6*x^6 + 433*a^4*x^4 + 36*a^2*x^2 + 3)*(a^2*x^2 + 1)^(3/2) + 2*(480*a^13*
x^13 + 2060*a^11*x^11 + 3496*a^9*x^9 + 2952*a^7*x^7 + 1283*a^5*x^5 + 274*a^
3*x^3 + 27*a*x)*(a^2*x^2 + 1) + (384*a^14*x^14 + 1976*a^12*x^12 + 4148*a^10
*x^10 + 4524*a^8*x^8 + 2699*a^6*x^6 + 842*a^4*x^4 + 111*a^2*x^2)*sqrt(a^2*x
^2 + 1))/((a^15*x^12 + 6*a^13*x^10 + 15*a^11*x^8 + (a^2*x^2 + 1)^3*a^9*x^6
+ 20*a^9*x^6 + 15*a^7*x^4 + 6*a^5*x^2 + 6*(a^10*x^7 + a^8*x^5)*(a^2*x^2 + 1
)^(5/2) + 15*(a^11*x^8 + 2*a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 20*(a
^12*x^9 + 3*a^10*x^7 + 3*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*
x^10 + 4*a^11*x^8 + 6*a^9*x^6 + 4*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 6*(a^1
4*x^11 + 5*a^12*x^9 + 10*a^10*x^7 + 10*a^8*x^5 + 5*a^6*x^3 + a^4*x)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**4,x)
```

```
[Out] Integral(x**3/asinh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x)^4, x)
```

3.69 $\int \frac{x^2}{\sinh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=138

$$-\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{24a^3}+\frac{9\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{3x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)}-\frac{x^2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{\sqrt{a^2x^2+1}}{3a^3\sinh^{-1}(ax)}-\frac{x}{3a^2\sinh^{-1}(ax)^2}-\frac{}{2\,\mathrm{s}}$$

```
[Out] -(x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - x/(3*a^2*ArcSinh[a*x]^2) -
x^3/(2*ArcSinh[a*x]^2) - Sqrt[1 + a^2*x^2]/(3*a^3*ArcSinh[a*x]) - (3*x^2*Sq
rt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]) - SinhIntegral[ArcSinh[a*x]]/(24*a^3) +
 (9*SinhIntegral[3*ArcSinh[a*x]])/(8*a^3)
```

Rubi [A] time = 0.305834, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5667, 5774, 5665, 3298, 5655, 5779}

$$-\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{24a^3}+\frac{9\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{3x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)}-\frac{x^2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{\sqrt{a^2x^2+1}}{3a^3\sinh^{-1}(ax)}-\frac{x}{3a^2\sinh^{-1}(ax)^2}-\frac{}{2\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^4,x]
```

```
[Out] -(x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - x/(3*a^2*ArcSinh[a*x]^2) -
x^3/(2*ArcSinh[a*x]^2) - Sqrt[1 + a^2*x^2]/(3*a^3*ArcSinh[a*x]) - (3*x^2*Sq
rt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]) - SinhIntegral[ArcSinh[a*x]]/(24*a^3) +
 (9*SinhIntegral[3*ArcSinh[a*x]])/(8*a^3)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^4}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{2\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{3a} + a\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} + \frac{3}{2}\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{\sinh^{-1}(ax)^2}\,dx}{3a^2}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\sinh^{-1}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)} + \frac{3\,\mathrm{Subst}}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\sinh^{-1}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)} + \frac{\mathrm{Subst}\big(}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\sinh^{-1}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)} - \frac{\mathrm{Shi}\big(\sin}{24}$$

Mathematica [A] time = 0.290157, size = 99, normalized size = 0.72

$$-\frac{\frac{4\left(2a^2x^2\sqrt{a^2x^2+1}+ax\left(3a^2x^2+2\right)\sinh^{-1}(ax)+\sqrt{a^2x^2+1}\left(9a^2x^2+2\right)\sinh^{-1}(ax)^2\right)}{\sinh^{-1}(ax)^3}+\mathrm{Shi}\left(\sinh^{-1}(ax)\right)-27\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x]^4,x]
```

```
[Out] -((4*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(2 + 3*a^2*x^2)*ArcSinh[a*x] + Sqrt
[1 + a^2*x^2]*(2 + 9*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + SinhIntegra
l[ArcSinh[a*x]] - 27*SinhIntegral[3*ArcSinh[a*x]])/(24*a^3)
```

Maple [A] time = 0.031, size = 115, normalized size = 0.8

$$\frac{1}{a^3}\left(\frac{1}{12\,(\mathrm{Arcsinh}\,(ax))^3}\sqrt{a^2x^2+1} + \frac{ax}{24\,(\mathrm{Arcsinh}\,(ax))^2} + \frac{1}{24\,\mathrm{Arcsinh}\,(ax)}\sqrt{a^2x^2+1} - \frac{\mathrm{Shi}\,(\mathrm{Arcsinh}\,(ax))}{24} - \frac{\cosh(3}{12\,(\mathrm{A}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^3*(1/12/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+1/24*a*x/arcsinh(a*x)^2+1/24/a
rcsinh(a*x)*(a^2*x^2+1)^(1/2)-1/24*Shi(arcsinh(a*x))-1/12/arcsinh(a*x)^3*co
sh(3*arcsinh(a*x))-1/8/arcsinh(a*x)^2*sinh(3*arcsinh(a*x))-3/8/arcsinh(a*x)
*cosh(3*arcsinh(a*x))+9/8*Shi(3*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (9*a^13*x^13 + 45*a^11*x^11 + 90*a^9*x^
9 + 90*a^7*x^7 + 45*a^5*x^5 + 9*a^3*x^3 + (9*a^8*x^8 + 13*a^6*x^6 + 3*a^4*x
^4 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (45*a^9*x^9 + 97*a^7*x^7 + 64*a^5*x^5 +
 10*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (90*a^10*x^10 + 258*a^8*x^8 + 264*a^
6*x^6 + 113*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1)^(3/2) + 2*(45*a^11*x^11
 + 161*a^9*x^9 + 219*a^7*x^7 + 141*a^5*x^5 + 44*a^3*x^3 + 6*a*x)*(a^2*x^2 +
 1) + (45*a^12*x^12 + 193*a^10*x^10 + 325*a^8*x^8 + 270*a^6*x^6 + 112*a^4*x
^4 + 19*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a
^10*x^10 + 13*a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*
a^11*x^11 + 17*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1)
 + (3*a^13*x^13 + 15*a^11*x^11 + 30*a^9*x^9 + 30*a^7*x^7 + 15*a^5*x^5 + 3*a
^3*x^3 + (3*a^8*x^8 + 4*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (15*a^9*x^
9 + 31*a^7*x^7 + 20*a^5*x^5 + 4*a^3*x^3)*(a^2*x^2 + 1)^2 + (30*a^10*x^10 +
84*a^8*x^8 + 84*a^6*x^6 + 35*a^4*x^4 + 5*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(
15*a^11*x^11 + 53*a^9*x^9 + 71*a^7*x^7 + 44*a^5*x^5 + 12*a^3*x^3 + a*x)*(a^
2*x^2 + 1) + (15*a^12*x^12 + 64*a^10*x^10 + 107*a^8*x^8 + 87*a^6*x^6 + 34*a
^4*x^4 + 5*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*
a^12*x^12 + 21*a^10*x^10 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*s
qrt(a^2*x^2 + 1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x^2 + 1)^(5/2)*a^8*x^5 +
10*a^9*x^6 + 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2
 + a^3 + 10*(a^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^11
*x^8 + 3*a^9*x^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 5*(a^12*x^9 + 4*a^1
0*x^7 + 6*a^8*x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^
2*x^2 + 1))^3) + integrate(1/6*(27*a^14*x^14 + 162*a^12*x^12 + 405*a^10*x^1
0 + 540*a^8*x^8 + 405*a^6*x^6 + 162*a^4*x^4 + (27*a^8*x^8 + 13*a^6*x^6 - 3*
a^4*x^4 + 3*a^2*x^2)*(a^2*x^2 + 1)^3 + 27*a^2*x^2 + (162*a^9*x^9 + 227*a^7*
x^7 + 63*a^5*x^5 - 3*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^(5/2) + (405*a^10*x^10
```

```
+ 940*a^8*x^8 + 687*a^6*x^6 + 143*a^4*x^4 - 21*a^2*x^2 - 12)*(a^2*x^2 + 1)^
2 + (540*a^11*x^11 + 1750*a^9*x^9 + 2058*a^7*x^7 + 1017*a^5*x^5 + 145*a^3*x
^3 - 24*a*x)*(a^2*x^2 + 1)^(3/2) + (405*a^12*x^12 + 1685*a^10*x^10 + 2727*a
^8*x^8 + 2118*a^6*x^6 + 782*a^4*x^4 + 123*a^2*x^2 + 12)*(a^2*x^2 + 1) + (16
2*a^13*x^13 + 823*a^11*x^11 + 1695*a^9*x^9 + 1790*a^7*x^7 + 1015*a^5*x^5 +
297*a^3*x^3 + 38*a*x)*sqrt(a^2*x^2 + 1))/((a^14*x^12 + 6*a^12*x^10 + 15*a^1
0*x^8 + (a^2*x^2 + 1)^3*a^8*x^6 + 20*a^8*x^6 + 15*a^6*x^4 + 6*a^4*x^2 + 6*(
a^9*x^7 + a^7*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^10*x^8 + 2*a^8*x^6 + a^6*x^4
)*(a^2*x^2 + 1)^2 + 20*(a^11*x^9 + 3*a^9*x^7 + 3*a^7*x^5 + a^5*x^3)*(a^2*x^
2 + 1)^(3/2) + 15*(a^12*x^10 + 4*a^10*x^8 + 6*a^8*x^6 + 4*a^6*x^4 + a^4*x^2
)*(a^2*x^2 + 1) + a^2 + 6*(a^13*x^11 + 5*a^11*x^9 + 10*a^9*x^7 + 10*a^7*x^5
 + 5*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\text{asinh}^4\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**4,x)
```

```
[Out] Integral(x**2/asinh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\text{arsinh}\,(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^4, x)
```

3.70 $\int \frac{x}{\sinh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=95

$$\frac{2\operatorname{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}$$

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - 1/(6*a^2*ArcSinh[a*x]^2) - x^
2/(3*ArcSinh[a*x]^2) - (2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) + (2*Cosh
Integral[2*ArcSinh[a*x]])/(3*a^2)
```

Rubi [A] time = 0.163369, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5667, 5774, 5665, 3301, 5675}

$$\frac{2\operatorname{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^4,x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - 1/(6*a^2*ArcSinh[a*x]^2) - x^
2/(3*ArcSinh[a*x]^2) - (2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) + (2*Cosh
Integral[2*ArcSinh[a*x]])/(3*a^2)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
```

```
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)^4}\,dx &= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}+\frac{\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{3a}+\frac{1}{3}(2a)\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}+\frac{2}{3}\int \frac{x}{\sinh^{-1}(ax)^2}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}-\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{3a^2}\\
&= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}-\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)}+\frac{2\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^2}
\end{aligned}
$$

Mathematica [A] time = 0.115911, size = 84, normalized size = 0.88

$$
-\frac{2ax\sqrt{a^2x^2+1}+4ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2+\left(2a^2x^2+1\right)\sinh^{-1}(ax)-4\sinh^{-1}(ax)^3\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{6a^2\sinh^{-1}(ax)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x]^4,x]
```

```
[Out] -(2*a*x*Sqrt[1 + a^2*x^2] + (1 + 2*a^2*x^2)*ArcSinh[a*x] + 4*a*x*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^2 - 4*ArcSinh[a*x]^3*CoshIntegral[2*ArcSinh[a*x]])/(6*
a^2*ArcSinh[a*x]^3)
```

Maple [A] time = 0.025, size = 60, normalized size = 0.6

$$\frac{1}{a^2}\left(-\frac{\sinh\left(2\,\mathrm{Arcsinh}\left(ax\right)\right)}{6\,\left(\mathrm{Arcsinh}\left(ax\right)\right)^3}-\frac{\cosh\left(2\,\mathrm{Arcsinh}\left(ax\right)\right)}{6\,\left(\mathrm{Arcsinh}\left(ax\right)\right)^2}-\frac{\sinh\left(2\,\mathrm{Arcsinh}\left(ax\right)\right)}{3\,\mathrm{Arcsinh}\left(ax\right)}+\frac{2\,\mathrm{Chi}\left(2\,\mathrm{Arcsinh}\left(ax\right)\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^2*(-1/6/arcsinh(a*x)^3*sinh(2*arcsinh(a*x))-1/6/arcsinh(a*x)^2*cosh(2*a
rcsinh(a*x))-1/3/arcsinh(a*x)*sinh(2*arcsinh(a*x))+2/3*Chi(2*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10*a^4*x^4 + 2
*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^8*x^8 + 9*a^6
*x^6 + 4*a^4*x^4)*(a^2*x^2 + 1)^2 + (4*a^12*x^12 + 20*a^10*x^10 + 40*a^8*x^
8 + 40*a^6*x^6 + 20*a^4*x^4 + 4*a^2*x^2 + 4*(a^7*x^7 + a^5*x^5)*(a^2*x^2 +
1)^(5/2) + (20*a^8*x^8 + 36*a^6*x^6 + 16*a^4*x^4 - 3*a^2*x^2 - 3)*(a^2*x^2
+ 1)^2 + (40*a^9*x^9 + 104*a^7*x^7 + 88*a^5*x^5 + 21*a^3*x^3 - 3*a*x)*(a^2*
x^2 + 1)^(3/2) + (40*a^10*x^10 + 136*a^8*x^8 + 168*a^6*x^6 + 91*a^4*x^4 + 2
2*a^2*x^2 + 3)*(a^2*x^2 + 1) + (20*a^11*x^11 + 84*a^9*x^9 + 136*a^7*x^7 + 1
07*a^5*x^5 + 42*a^3*x^3 + 7*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))^2 + 4*(5*a^9*x^9 + 13*a^7*x^7 + 11*a^5*x^5 + 3*a^3*x^3)*(a^2*x^2 + 1)
^(3/2) + 4*(5*a^10*x^10 + 17*a^8*x^8 + 21*a^6*x^6 + 11*a^4*x^4 + 2*a^2*x^2)
*(a^2*x^2 + 1) + (2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10
*a^4*x^4 + 2*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + (10*a^8*
```

```
x^8 + 18*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^2 + (20*a^9*x^9 + 52*
a^7*x^7 + 47*a^5*x^5 + 17*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1)^(3/2) + (20*a^10*x
^10 + 68*a^8*x^8 + 87*a^6*x^6 + 51*a^4*x^4 + 13*a^2*x^2 + 1)*(a^2*x^2 + 1)
+ (10*a^11*x^11 + 42*a^9*x^9 + 69*a^7*x^7 + 55*a^5*x^5 + 21*a^3*x^3 + 3*a*x
)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^11*x^11 + 21*a^9
*x^9 + 34*a^7*x^7 + 26*a^5*x^5 + 9*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^12
*x^10 + 5*a^10*x^8 + (a^2*x^2 + 1)^(5/2)*a^7*x^5 + 10*a^8*x^6 + 10*a^6*x^4
+ 5*a^4*x^2 + 5*(a^8*x^6 + a^6*x^4)*(a^2*x^2 + 1)^2 + 10*(a^9*x^7 + 2*a^7*x
^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^10*x^8 + 3*a^8*x^6 + 3*a^6*x^4 +
a^4*x^2)*(a^2*x^2 + 1) + a^2 + 5*(a^11*x^9 + 4*a^9*x^7 + 6*a^7*x^5 + 4*a^5*
x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate
(1/6*(8*a^13*x^13 + 48*a^11*x^11 + 120*a^9*x^9 + 8*(a^2*x^2 + 1)^3*a^7*x^7
+ 160*a^7*x^7 + 120*a^5*x^5 + 48*a^3*x^3 + (48*a^8*x^8 + 48*a^6*x^6 + 4*a^4
*x^4 + 12*a^2*x^2 + 15)*(a^2*x^2 + 1)^(5/2) + 8*(15*a^9*x^9 + 30*a^7*x^7 +
17*a^5*x^5 + 5*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^2 + 2*(80*a^10*x^10 + 240*a^8
*x^8 + 252*a^6*x^6 + 104*a^4*x^4 + 3*a^2*x^2 - 9)*(a^2*x^2 + 1)^(3/2) + 8*(
15*a^11*x^11 + 60*a^9*x^9 + 92*a^7*x^7 + 63*a^5*x^5 + 15*a^3*x^3 - a*x)*(a^
2*x^2 + 1) + 8*a*x + (48*a^12*x^12 + 240*a^10*x^10 + 484*a^8*x^8 + 484*a^6*
x^6 + 243*a^4*x^4 + 58*a^2*x^2 + 7)*sqrt(a^2*x^2 + 1))/((a^13*x^12 + 6*a^11
*x^10 + 15*a^9*x^8 + (a^2*x^2 + 1)^3*a^7*x^6 + 20*a^7*x^6 + 15*a^5*x^4 + 6*
a^3*x^2 + 6*(a^8*x^7 + a^6*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^9*x^8 + 2*a^7*x
^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 20*(a^10*x^9 + 3*a^8*x^7 + 3*a^6*x^5 + a^4*
x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^11*x^10 + 4*a^9*x^8 + 6*a^7*x^6 + 4*a^5*x^
4 + a^3*x^2)*(a^2*x^2 + 1) + 6*(a^12*x^11 + 5*a^10*x^9 + 10*a^8*x^7 + 10*a^
6*x^5 + 5*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 +
1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**4,x)
```

```
[Out] Integral(x/asinh(a*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^4, x)
```

3.71 $\int \frac{1}{\sinh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}+\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{6a}-\frac{x}{6\sinh^{-1}(ax)^2}$$

```
[Out] -Sqrt[1 + a^2*x^2]/(3*a*ArcSinh[a*x]^3) - x/(6*ArcSinh[a*x]^2) - Sqrt[1 + a
^2*x^2]/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]]/(6*a)
```

Rubi [A] time = 0.149746, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5655, 5774, 5779, 3298}

$$-\frac{\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}+\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{6a}-\frac{x}{6\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-4),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*a*ArcSinh[a*x]^3) - x/(6*ArcSinh[a*x]^2) - Sqrt[1 + a
^2*x^2]/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]]/(6*a)
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sinh^{-1}(ax)^4}\,dx &= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}+\frac{1}{3}a\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{x}{6\sinh^{-1}(ax)^2}+\frac{1}{6}\int \frac{1}{\sinh^{-1}(ax)^2}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{x}{6\sinh^{-1}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)}+\frac{1}{6}a\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{x}{6\sinh^{-1}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)}+\frac{\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{6a}\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3}-\frac{x}{6\sinh^{-1}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)}+\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{6a}
\end{aligned}
$$

Mathematica [A] time = 0.0584567, size = 69, normalized size = 0.91

$$
-\frac{2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2+\sinh^{-1}(ax)^3\left(-\mathbf{Shi}\left(\sinh^{-1}(ax)\right)\right)+ax\sinh^{-1}(ax)}{6a\sinh^{-1}(ax)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-4),x]
```

```
[Out] -(2*Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2
 - ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x]])/(6*a*ArcSinh[a*x]^3)
```

Maple [A] time = 0.023, size = 61, normalized size = 0.8

$$\frac{1}{a}\left(-\frac{1}{3\left(\operatorname{Arcsinh}(ax)\right)^{3}}\sqrt{a^{2}x^{2}+1}-\frac{ax}{6\left(\operatorname{Arcsinh}(ax)\right)^{2}}-\frac{1}{6\operatorname{Arcsinh}(ax)}\sqrt{a^{2}x^{2}+1}+\frac{\operatorname{Shi}(\operatorname{Arcsinh}(ax))}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^4,x)
```

```
[Out] 1/a*(-1/3/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-1/6*a*x/arcsinh(a*x)^2-1/6/arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)+1/6*Shi(arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^11*x^11 + 10*a^9*x^9 + 20*a^7*x^7 + 20*a^5*x^5 + 10*a^3*x^3 + 2*(
a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^7*x^7 + 9*a^5*x^5 + 4*a^3*x
^3)*(a^2*x^2 + 1)^2 + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*x^7 + 10*a^5*x^5 + 5*
a^3*x^3 + (a^6*x^6 + a^4*x^4 + 3*a^2*x^2 + 3)*(a^2*x^2 + 1)^(5/2) + (5*a^7*
x^7 + 9*a^5*x^5 + 10*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 + 26*a^
6*x^6 + 22*a^4*x^4 + 3*a^2*x^2 - 3)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^9 + 17
*a^7*x^7 + 18*a^5*x^5 + 5*a^3*x^3 - a*x)*(a^2*x^2 + 1) + a*x + (5*a^10*x^10
 + 21*a^8*x^8 + 31*a^6*x^6 + 20*a^4*x^4 + 6*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))
*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^8*x^8 + 13*a^6*x^6 + 11*a^4*x^4 +
3*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^9*x^9 + 17*a^7*x^7 + 21*a^5*x^5 + 1
1*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + 2*a*x + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*
x^7 + 10*a^5*x^5 + 5*a^3*x^3 + (a^6*x^6 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5
*a^7*x^7 + 5*a^5*x^5 - 2*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 + 2
0*a^6*x^6 + 10*a^4*x^4 - a^2*x^2 - 1)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^9 +
15*a^7*x^7 + 16*a^5*x^5 + 7*a^3*x^3 + a*x)*(a^2*x^2 + 1) + a*x + (5*a^10*x^
10 + 20*a^8*x^8 + 31*a^6*x^6 + 23*a^4*x^4 + 8*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1
))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^10*x^10 + 21*a^8*x^8 + 34*a^6*x^6
+ 26*a^4*x^4 + 9*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^11*x^10 + 5*a^9*x^8 +
(a^2*x^2 + 1)^(5/2)*a^6*x^5 + 10*a^7*x^6 + 10*a^5*x^4 + 5*a^3*x^2 + 5*(a^7*
x^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 10*(a^8*x^7 + 2*a^6*x^5 + a^4*x^3)*(a^2*x^
```

```
2 + 1)^(3/2) + 10*(a^9*x^8 + 3*a^7*x^6 + 3*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1)
 + 5*(a^10*x^9 + 4*a^8*x^7 + 6*a^6*x^5 + 4*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 +
1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate(1/6*(a^12*x^12 + 6*a^10
*x^10 + 15*a^8*x^8 + 20*a^6*x^6 + 15*a^4*x^4 + (a^6*x^6 - a^4*x^4 - 9*a^2*x
^2 - 15)*(a^2*x^2 + 1)^3 + 6*a^2*x^2 + (6*a^7*x^7 + a^5*x^5 - 31*a^3*x^3 -
33*a*x)*(a^2*x^2 + 1)^(5/2) + (15*a^8*x^8 + 20*a^6*x^6 - 19*a^4*x^4 - 3*a^2
*x^2 + 21)*(a^2*x^2 + 1)^2 + (20*a^9*x^9 + 50*a^7*x^7 + 54*a^5*x^5 + 59*a^3
*x^3 + 35*a*x)*(a^2*x^2 + 1)^(3/2) + (15*a^10*x^10 + 55*a^8*x^8 + 101*a^6*x
^6 + 90*a^4*x^4 + 22*a^2*x^2 - 7)*(a^2*x^2 + 1) + (6*a^11*x^11 + 29*a^9*x^9
 + 65*a^7*x^7 + 66*a^5*x^5 + 23*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^1
2*x^12 + 6*a^10*x^10 + 15*a^8*x^8 + (a^2*x^2 + 1)^3*a^6*x^6 + 20*a^6*x^6 +
15*a^4*x^4 + 6*a^2*x^2 + 6*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^
8*x^8 + 2*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^2 + 20*(a^9*x^9 + 3*a^7*x^7 + 3*
a^5*x^5 + a^3*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^10*x^10 + 4*a^8*x^8 + 6*a^6*
x^6 + 4*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 6*(a^11*x^11 + 5*a^9*x^9 + 10*a^
7*x^7 + 10*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt
(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\text{arsinh}\left(ax\right)^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}^{4}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**4,x)
```

```
[Out] Integral(asinh(a*x)**(-4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-4), x)
```

3.72 $\int \frac{1}{x \sinh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)^4}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]^4), x]
```

Rubi [A] time = 0.0127958, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^4} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^4} \, dx$$

Mathematica [A] time = 1.80128, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^4),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]^4), x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\operatorname{Arcsinh}\left(ax\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^4,x)
```

```
[Out] int(1/x/arcsinh(a*x)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (4*(a^6*x^6 + 3*a^4*x^4 + 2*a^2*x^2)*(a
^2*x^2 + 1)^(5/2) + (16*a^7*x^7 + 46*a^5*x^5 + 37*a^3*x^3 + 7*a*x)*(a^2*x^2
 + 1)^2 + (24*a^8*x^8 + 66*a^6*x^6 + 59*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2
+ 1)^(3/2) + (16*a^9*x^9 + 42*a^7*x^7 + 39*a^5*x^5 + 16*a^3*x^3 + 3*a*x)*(a
^2*x^2 + 1) + (4*a^10*x^10 + 10*a^8*x^8 + 9*a^6*x^6 + 4*a^4*x^4 + a^2*x^2)*
sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10 + 13*a^8
*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^11 + 17*a^
9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (2*(a^6*x^6 +
a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (8*a^7*x^7 + 13*a^5*x^5 + 5*a^3*x^3)*(a^2*x^
2 + 1)^2 + (12*a^8*x^8 + 27*a^6*x^6 + 19*a^4*x^4 + 4*a^2*x^2)*(a^2*x^2 + 1)
^(3/2) + (8*a^9*x^9 + 23*a^7*x^7 + 23*a^5*x^5 + 9*a^3*x^3 + a*x)*(a^2*x^2 +
 1) + (2*a^10*x^10 + 7*a^8*x^8 + 9*a^6*x^6 + 5*a^4*x^4 + a^2*x^2)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10 + 34
*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^13*x^13
 + 5*a^11*x^11 + (a^2*x^2 + 1)^(5/2)*a^8*x^8 + 10*a^9*x^9 + 10*a^7*x^7 + 5*
a^5*x^5 + a^3*x^3 + 5*(a^9*x^9 + a^7*x^7)*(a^2*x^2 + 1)^2 + 10*(a^10*x^10 +
 2*a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^11 + 3*a^9*x^9 + 3*a
^7*x^7 + a^5*x^5)*(a^2*x^2 + 1) + 5*(a^12*x^12 + 4*a^10*x^10 + 6*a^8*x^8 +
4*a^6*x^6 + a^4*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) - i
```

```
ntegrate(1/6*(8*(a^7*x^7 + 6*a^5*x^5 + 6*a^3*x^3)*(a^2*x^2 + 1)^3 + (40*a^8
*x^8 + 204*a^6*x^6 + 228*a^4*x^4 + 57*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + 2*(40*
a^9*x^9 + 168*a^7*x^7 + 200*a^5*x^5 + 87*a^3*x^3 + 15*a*x)*(a^2*x^2 + 1)^2
+ 2*(40*a^10*x^10 + 132*a^8*x^8 + 156*a^6*x^6 + 91*a^4*x^4 + 30*a^2*x^2 + 3
)*(a^2*x^2 + 1)^(3/2) + 2*(20*a^11*x^11 + 48*a^9*x^9 + 48*a^7*x^7 + 35*a^5*
x^5 + 18*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + (8*a^12*x^12 + 12*a^10*x^10 + 4*a
^8*x^8 + 5*a^6*x^6 + 6*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^15*x^16 +
6*a^13*x^14 + 15*a^11*x^12 + (a^2*x^2 + 1)^3*a^9*x^10 + 20*a^9*x^10 + 15*a^
7*x^8 + 6*a^5*x^6 + a^3*x^4 + 6*(a^10*x^11 + a^8*x^9)*(a^2*x^2 + 1)^(5/2) +
 15*(a^11*x^12 + 2*a^9*x^10 + a^7*x^8)*(a^2*x^2 + 1)^2 + 20*(a^12*x^13 + 3*
a^10*x^11 + 3*a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*x^14 + 4*a^
11*x^12 + 6*a^9*x^10 + 4*a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1) + 6*(a^14*x^15 +
5*a^12*x^13 + 10*a^10*x^11 + 10*a^8*x^9 + 5*a^6*x^7 + a^4*x^5)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \operatorname{arsinh}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(a*x)^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{asinh}^4(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**4,x)
```

```
[Out] Integral(1/(x*asinh(a*x)**4), x)
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(ax)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^4), x)
```

3.73 $\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \sinh^{-1}(ax)^4}, x\right)$$

[Out] Unintegrable[1/(x^2*ArcSinh[a*x]^4), x]

Rubi [A] time = 0.0141296, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*ArcSinh[a*x]^4),x]

[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^4), x]

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx = \int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$$

Mathematica [A] time = 8.82175, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2*ArcSinh[a*x]^4),x]

[Out] Integrate[1/(x^2*ArcSinh[a*x]^4), x]

Maple [A] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\operatorname{Arcsinh}\left(ax\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^4,x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (a^13*x^13 + 5*a^11*x^11 + 10*a^9*x^9 +
 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 + 13*a^6*x^6 + 27*a^4*x^4 + 15
*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 57*a^7*x^7 + 124*a^5*x^5 + 90*
a^3*x^3 + 18*a*x)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 98*a^8*x^8 + 220*a^6*x^
6 + 189*a^4*x^4 + 63*a^2*x^2 + 6)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^11 + 41
*a^9*x^9 + 93*a^7*x^7 + 89*a^5*x^5 + 38*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1) + (5
*a^12*x^12 + 33*a^10*x^10 + 73*a^8*x^8 + 74*a^6*x^6 + 36*a^4*x^4 + 7*a^2*x^
2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10 + 13*
a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^11 + 17
*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (a^13*x^13
+ 5*a^11*x^11 + 10*a^9*x^9 + 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 +
4*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 21*a^7*x^7 + 24*a
^5*x^5 + 8*a^3*x^3)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 44*a^8*x^8 + 64*a^6*x
^6 + 37*a^4*x^4 + 7*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^11 + 23*a^9*
x^9 + 39*a^7*x^7 + 30*a^5*x^5 + 10*a^3*x^3 + a*x)*(a^2*x^2 + 1) + (5*a^12*x
^12 + 24*a^10*x^10 + 45*a^8*x^8 + 41*a^6*x^6 + 18*a^4*x^4 + 3*a^2*x^2)*sqrt
(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10
 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^13
*x^14 + 5*a^11*x^12 + (a^2*x^2 + 1)^(5/2)*a^8*x^9 + 10*a^9*x^10 + 10*a^7*x^
```

```
8 + 5*a^5*x^6 + a^3*x^4 + 5*(a^9*x^10 + a^7*x^8)*(a^2*x^2 + 1)^2 + 10*(a^10
*x^11 + 2*a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^12 + 3*a^9*x^
10 + 3*a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1) + 5*(a^12*x^13 + 4*a^10*x^11 + 6*a^
8*x^9 + 4*a^6*x^7 + a^4*x^5)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)
)^3) - integrate(1/6*(a^15*x^15 + 6*a^13*x^13 + 15*a^11*x^11 + 20*a^9*x^9 +
 15*a^7*x^7 + 6*a^5*x^5 + a^3*x^3 + (a^9*x^9 + 39*a^7*x^7 + 135*a^5*x^5 + 1
05*a^3*x^3)*(a^2*x^2 + 1)^3 + (6*a^10*x^10 + 201*a^8*x^8 + 677*a^6*x^6 + 66
3*a^4*x^4 + 174*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (15*a^11*x^11 + 420*a^9*x^9
+ 1373*a^7*x^7 + 1565*a^5*x^5 + 705*a^3*x^3 + 108*a*x)*(a^2*x^2 + 1)^2 + (2
0*a^12*x^12 + 450*a^10*x^10 + 1422*a^8*x^8 + 1787*a^6*x^6 + 1059*a^4*x^4 +
288*a^2*x^2 + 24)*(a^2*x^2 + 1)^(3/2) + (15*a^13*x^13 + 255*a^11*x^11 + 773
*a^9*x^9 + 1026*a^7*x^7 + 714*a^5*x^5 + 257*a^3*x^3 + 36*a*x)*(a^2*x^2 + 1)
 + (6*a^14*x^14 + 69*a^12*x^12 + 197*a^10*x^10 + 266*a^8*x^8 + 201*a^6*x^6
+ 83*a^4*x^4 + 14*a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^15*x^17 + 6*a^13*x^15 + 1
5*a^11*x^13 + (a^2*x^2 + 1)^3*a^9*x^11 + 20*a^9*x^11 + 15*a^7*x^9 + 6*a^5*x
^7 + a^3*x^5 + 6*(a^10*x^12 + a^8*x^10)*(a^2*x^2 + 1)^(5/2) + 15*(a^11*x^13
 + 2*a^9*x^11 + a^7*x^9)*(a^2*x^2 + 1)^2 + 20*(a^12*x^14 + 3*a^10*x^12 + 3*
a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*x^15 + 4*a^11*x^13 + 6*a
^9*x^11 + 4*a^7*x^9 + a^5*x^7)*(a^2*x^2 + 1) + 6*(a^14*x^16 + 5*a^12*x^14 +
 10*a^10*x^12 + 10*a^8*x^10 + 5*a^6*x^8 + a^4*x^6)*sqrt(a^2*x^2 + 1))*log(a
*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \operatorname{arsinh}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arcsinh(a*x)^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{asinh}^4(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**4,x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)**4), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)^4), x)
```

3.74 $\int x^4 \sqrt{\sinh^{-1}(ax)}\, dx$

Optimal. Leaf size=182

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}+\frac{\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{320a^5}-\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{3}}\text{Erfi}}{}$$

```
[Out] (x^5*Sqrt[ArcSinh[a*x]])/5 + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(32*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) + (Sqrt[Pi/5]*Erf[Sqr
t[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(
32*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) - (Sqrt[Pi
/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5)
```

Rubi [A] time = 0.324456, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}+\frac{\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{320a^5}-\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{3}}\text{Erfi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x^5*Sqrt[ArcSinh[a*x]])/5 + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(32*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) + (Sqrt[Pi/5]*Erf[Sqr
t[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(
32*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) - (Sqrt[Pi
/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^4\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} - \frac{1}{10}a\int \frac{x^5}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{i\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8\sqrt{x}} - \frac{5i\sinh(3x)}{16\sqrt{x}} + \frac{i\sinh(5x)}{16\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{160a^5} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a^5}$$

$$= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{320a^5} - \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{320a^5} - \text{Subs}$$

$$= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{160a^5} - \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{160a^5} -$$

$$= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{64a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}\right.}{320a^5}$$

Mathematica [A] time = 0.0446512, size = 161, normalized size = 0.88

$$\frac{\frac{\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -5\sinh^{-1}(ax)\right)}{160\sqrt{5}\sqrt{-\sinh^{-1}(ax)}} - \frac{\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -3\sinh^{-1}(ax)\right)}{32\sqrt{3}\sqrt{-\sinh^{-1}(ax)}} + \frac{\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -\sinh^{-1}(ax)\right)}{16\sqrt{-\sinh^{-1}(ax)}} - \frac{1}{16}\text{Gamma}\left(\frac{3}{2}, \text{sinh}\right.}{a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -5*ArcSinh[a*x]])/(160*Sqrt[5]*Sqrt[-ArcSin
h[a*x]]) - (Sqrt[ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]])/(32*Sqrt[3]*Sqr
t[-ArcSinh[a*x]]) + (Sqrt[ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/(16*Sqrt
[-ArcSinh[a*x]]) - Gamma[3/2, ArcSinh[a*x]]/16 + Gamma[3/2, 3*ArcSinh[a*x]]
/(32*Sqrt[3]) - Gamma[3/2, 5*ArcSinh[a*x]]/(160*Sqrt[5]))/a^5
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int x^4\sqrt{\text{Arcsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{\operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(asinh(a*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*sqrt(arcsinh(a*x)), x)
```

3.75 $\int x^3\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{\pi}\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\pi}\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{3\sqrt{s}}{}$$

```
[Out] (-3*Sqrt[ArcSinh[a*x]])/(32*a^4) + (x^4*Sqrt[ArcSinh[a*x]])/4 - (Sqrt[Pi]*E
rf[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(32*a^4) - (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[
Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(32*a^4)
```

Rubi [A] time = 0.261595, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\pi}\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{3\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcSinh[a*x]],x]
```


```
[Out] (-3*Sqrt[ArcSinh[a*x]])/(32*a^4) + (x^4*Sqrt[ArcSinh[a*x]])/4 - (Sqrt[Pi]*E
rf[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(32*a^4) - (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[
Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(32*a^4)
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^3\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{1}{8}a\int \frac{x^4}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^4}$$

$$= \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{8a^4}$$

$$= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{64a^4} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,d\right.}{16a}$$

$$= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{128a^4} - \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \sinh\right.}{128a^4}$$

$$= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{64a^4} - \frac{\text{Subst}\left(\int e^{4x^2}\,dx, x\right.}{64a^4}$$

$$= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{}}{}$$

Mathematica [A] time = 0.0357477, size = 101, normalized size = 0.73

$$\frac{\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -4\sinh^{-1}(ax)\right) - 4\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -2\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(\text{Gamma}\left(\frac{3}{2}\right.\right.}{128a^4\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]] - 4*Sqrt[2]*Sqrt[ArcSinh[a*
x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(-4*Sqrt[2]*Gamma[3/2
, 2*ArcSinh[a*x]] + Gamma[3/2, 4*ArcSinh[a*x]]))/(128*a^4*Sqrt[-ArcSinh[a*x
]])
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int x^3\sqrt{\text{Arcsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(arcsinh(a*x)), x)
```

3.76 $\int x^2\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=120

$$-\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}+\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\text{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}+\frac{1}{3}x^3\sqrt{}$$

```
[Out] (x^3*Sqrt[ArcSinh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^3) +
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(48*a^3) + (Sqrt[Pi]*Erfi[Sqrt
[ArcSinh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(
48*a^3)
```

Rubi [A] time = 0.244318, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}+\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\text{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}+\frac{1}{3}x^3\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x^3*Sqrt[ArcSinh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^3) +
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(48*a^3) + (Sqrt[Pi]*Erfi[Sqrt
[ArcSinh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(
48*a^3)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{1}{6}a\int \frac{x^3}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{6a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{x}} - \frac{i\sinh(3x)}{4\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{6a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{24a^3} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{48a^3} - \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{48a^3} - \frac{\text{Subs}}{}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{24a^3} - \frac{\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{24a^3} -$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3} + \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax\right.}\right.}{16a^3}$$

Mathematica [A] time = 0.0344497, size = 101, normalized size = 0.84

$$\frac{\sqrt{3}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2}, -3\sinh^{-1}(ax)\right) - 9\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2}, -\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(9\text{Gamma}\left(\right.\right.}{72a^3\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]] - 9*Sqrt[ArcSinh[a*
x]]*Gamma[3/2, -ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(9*Gamma[3/2, ArcSinh[a
*x]] - Sqrt[3]*Gamma[3/2, 3*ArcSinh[a*x]]))/(72*a^3*Sqrt[-ArcSinh[a*x]])
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x^2\sqrt{\text{Arcsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arcsinh(a*x)), x)
```

3.77 $\int x\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}+\frac{\sqrt{\sinh^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)}$$

```
[Out] Sqrt[ArcSinh[a*x]]/(4*a^2) + (x^2*Sqrt[ArcSinh[a*x]])/2 - (Sqrt[Pi/2]*Erf[S
qrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(16*a^2)
```

Rubi [A] time = 0.19105, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}+\frac{\sqrt{\sinh^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Sqrt[ArcSinh[a*x]]/(4*a^2) + (x^2*Sqrt[ArcSinh[a*x]])/2 - (Sqrt[Pi/2]*Erf[S
qrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(16*a^2)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{\sinh^{-1}(ax)}\,dx &= \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{1}{4}a\int \frac{x^2}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx \\
&= \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^2} \\
&= \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{4a^2} \\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^2} \\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^2} - \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(\right.}{16a^2} \\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a^2} - \frac{\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{s}\right.}{8a^2} \\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0325835, size = 52, normalized size = 0.56

$$
\frac{\frac{\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2}, -2\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \text{Gamma}\left(\frac{3}{2}, 2\sinh^{-1}(ax)\right)}{8\sqrt{2}a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Gam
ma[3/2, 2*ArcSinh[a*x]])/(8*Sqrt[2]*a^2)
```

Maple [A] time = 0.105, size = 75, normalized size = 0.8

$$
\frac{\sqrt{2}}{32\sqrt{\pi}a^2}\left(8\sqrt{2}\sqrt{\text{Arcsinh}(ax)}\sqrt{\pi}x^2a^2 + 4\sqrt{2}\sqrt{\text{Arcsinh}(ax)}\sqrt{\pi} - \pi\,\text{Erf}\left(\sqrt{2}\sqrt{\text{Arcsinh}(ax)}\right) - \pi\,\text{erfi}\left(\sqrt{2}\sqrt{\text{Arcsinh}(a\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^(1/2),x)
```

```
[Out] 1/32*2^(1/2)*(8*2^(1/2)*arcsinh(a*x)^(1/2)*Pi^(1/2)*x^2*a^2+4*2^(1/2)*arcsi
nh(a*x)^(1/2)*Pi^(1/2)-Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-Pi*erfi(2^(1/2)*a
rcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arcsinh(a*x)), x)
```

3.78 $\int \sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\pi}\mathbf{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}-\frac{\sqrt{\pi}\mathbf{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}+x\sqrt{\sinh^{-1}(ax)}$$

```
[Out] x*Sqrt[ArcSinh[a*x]] + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(4*a) - (Sqrt[Pi]
*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a)
```

Rubi [A] time = 0.107191, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\mathbf{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}-\frac{\sqrt{\pi}\mathbf{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}+x\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[a*x]],x]
```

```
[Out] x*Sqrt[ArcSinh[a*x]] + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(4*a) - (Sqrt[Pi]
*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a)
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sinh^{-1}(ax)}\,dx &= x\sqrt{\sinh^{-1}(ax)} - \frac{1}{2}a\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx \\
&= x\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a} \\
&= x\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a} \\
&= x\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a} - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a} \\
&= x\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0467143, size = 45, normalized size = 0.85

$$-\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{3}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}}+\mathrm{Gamma}\left(\frac{3}{2},\sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -((Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gamm
a[3/2, ArcSinh[a*x]])/(2*a)
```

Maple [A] time = 0.068, size = 42, normalized size = 0.8

$$\frac{1}{4\sqrt{\pi}a}\left(4\sqrt{\mathrm{Arcsinh}\,(ax)}\sqrt{\pi}xa+\pi\,\mathrm{Erf}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)-\pi\,\mathrm{erfi}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2),x)
```

```
[Out] 1/4*(4*arcsinh(a*x)^(1/2)*Pi^(1/2)*x*a+Pi*erf(arcsinh(a*x)^(1/2))-Pi*erfi(a
rcsinh(a*x)^(1/2)))/Pi^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x)), x)
```

3.79 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sqrt{\sinh^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcSinh[a*x]]/x, x]
```

Rubi [A] time = 0.0136434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[ArcSinh[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt{\sinh^{-1}(ax)}}{x}\,dx$$

Mathematica [A] time = 0.33273, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/x,x]
```

```
[Out] Integrate[Sqrt[ArcSinh[a*x]]/x, x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\operatorname{Arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/x,x)
```

```
[Out] int(arcsinh(a*x)^(1/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(asinh(a*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/x, x)
```

3.80 $\int x^4 \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=330

$$\frac{3\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}-\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{200a^5}+\frac{3\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}+$$

```
[Out] (-4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(25*a^5) + (2*x^2*Sqrt[1 + a^2*x^
2]*Sqrt[ArcSinh[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x
]])/(50*a) + (x^5*ArcSinh[a*x]^(3/2))/5 + (3*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]
]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (3*
Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erf[
Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x
]]])/(64*a^5) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (
3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*E
rfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5)
```

Rubi [A] time = 0.712764, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5657, 3307, 2180, 2204, 2205, 5669, 5448}

$$\frac{3\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}-\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}-\frac{\sqrt{\frac{\pi}{3}}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{200a^5}+\frac{3\sqrt{\frac{\pi}{5}}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^(3/2),x]
```


```
[Out] (-4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(25*a^5) + (2*x^2*Sqrt[1 + a^2*x^
2]*Sqrt[ArcSinh[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x
]])/(50*a) + (x^5*ArcSinh[a*x]^(3/2))/5 + (3*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]
]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (3*
Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erf[
Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x
]]])/(64*a^5) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (
3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*E
rfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5)
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^{3/2} - \frac{1}{10}(3a)\int \frac{x^5\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5\sinh^{-1}(ax)^{3/2} + \frac{3}{100}\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{6\int \frac{x^3\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}}{25a}$$

$$= \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5$$

Mathematica [A] time = 0.11715, size = 152, normalized size = 0.46

$$\frac{\frac{9\sqrt{5}\sqrt{-\sinh^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},-5\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{125\sqrt{3}\sqrt{\sinh^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \frac{2250\sqrt{-\sinh^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - 2250}{36000a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((9*Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a
*x]] + (125*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]])/Sqrt[-A
```

```
rcSinh[a*x]] + (2250*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[Ar
cSinh[a*x]] - 2250*Gamma[5/2, ArcSinh[a*x]] + 125*Sqrt[3]*Gamma[5/2, 3*ArcS
inh[a*x]] - 9*Sqrt[5]*Gamma[5/2, 5*ArcSinh[a*x]])/(36000*a^5)
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int x^4 (\text{Arcsinh}(ax))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

<u>**Sympy**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(3/2), x)
```

3.81 $\int x^3 \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=199

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{128a^4}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{128a^4}-$$

```
[Out] (9*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 + a^2*x
^2]*Sqrt[ArcSinh[a*x]])/(32*a) - (3*ArcSinh[a*x]^(3/2))/(32*a^4) + (x^4*Arc
Sinh[a*x]^(3/2))/4 - (3*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(2048*a^4) + (3
*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(128*a^4) + (3*Sqrt[Pi]*Erfi[2
*Sqrt[ArcSinh[a*x]]])/(2048*a^4) - (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(128*a^4)
```

Rubi [A] time = 0.487372, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{128a^4}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{128a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^(3/2),x]
```


```
[Out] (9*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 + a^2*x
^2]*Sqrt[ArcSinh[a*x]])/(32*a) - (3*ArcSinh[a*x]^(3/2))/(32*a^4) + (x^4*Arc
Sinh[a*x]^(3/2))/4 - (3*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(2048*a^4) + (3
*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(128*a^4) + (3*Sqrt[Pi]*Erfi[2
*Sqrt[ArcSinh[a*x]]])/(2048*a^4) - (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(128*a^4)
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} - \frac{1}{8}(3a)\int \frac{x^4\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} + \frac{3}{64}\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{9\int \frac{x^2\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,d\dots}{32a}$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(\dots}{\dots}\right.}{\dots}$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} \dots$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} \dots$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} \dots$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} \dots$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} \dots$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} \dots$$

Mathematica [A] time = 0.0354882, size = 102, normalized size = 0.51

$$\frac{-\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-4\sinh^{-1}(ax)\right)+8\sqrt{2}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-2\sinh^{-1}(ax)\right)+\sqrt{-\sinh^{-1}(ax)}\left(\text{Gamma}\right.}{512a^4\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-(Sqrt[ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]]) + 8*Sqrt[2]*Sqrt[ArcSinh
[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(-8*Sqrt[2]*Gamma[
5/2, 2*ArcSinh[a*x]] + Gamma[5/2, 4*ArcSinh[a*x]]))/(512*a^4*Sqrt[-ArcSinh[
a*x]])
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x^3 (\text{Arcsinh}(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(3/2), x)
```

3.82 $\int x^2 \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=179

$$-\frac{3\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}-\frac{3\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}-\frac{x^2}{}$$

```
[Out] (Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*Sqr
t[ArcSinh[a*x]])/(6*a) + (x^3*ArcSinh[a*x]^(3/2))/3 - (3*Sqrt[Pi]*Erf[Sqrt[
ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(96
*a^3) - (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erfi[S
qrt[3]*Sqrt[ArcSinh[a*x]]])/(96*a^3)
```

Rubi [A] time = 0.371351, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5657, 3307, 2180, 2204, 2205, 5669, 5448}

$$-\frac{3\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}-\frac{3\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}-\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*Sqr
t[ArcSinh[a*x]])/(6*a) + (x^3*ArcSinh[a*x]^(3/2))/3 - (3*Sqrt[Pi]*Erf[Sqrt[
ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(96
*a^3) - (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erfi[S
qrt[3]*Sqrt[ArcSinh[a*x]]])/(96*a^3)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{1}{2}a\int \frac{x^3\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{1}{12}\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{\int \frac{x\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{3a}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int \frac{\cosh(x)\sin}{\sqrt{x}}\right.}{}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x}{4\sqrt{x}}\right.\right.}{}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,d\right.}{48a}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x\right.}{96a^3}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(a}\right.}{12a^3}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}}\right.}{32a^3}$$

Mathematica [A] time = 0.0339384, size = 102, normalized size = 0.57

$$\frac{-\sqrt{3}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-3\sinh^{-1}(ax)\right)+27\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{5}{2},-\sinh^{-1}(ax)\right)+\sqrt{-\sinh^{-1}(ax)}\left(27\text{Gamm}\right.}{216a^3\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-(Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]]) + 27*Sqrt[ArcSin
h[a*x]]*Gamma[5/2, -ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(27*Gamma[5/2, ArcS
inh[a*x]] - Sqrt[3]*Gamma[5/2, 3*ArcSinh[a*x]]))/(216*a^3*Sqrt[-ArcSinh[a*x
]])
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x^2 (\text{Arcsinh}(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2*asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^(3/2), x)
```

3.83 $\int x \sinh^{-1}(ax)^{3/2}\, dx$

Optimal. Leaf size=122

$$-\frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}-\frac{3x\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{8a}+\frac{\sinh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\sinh^{-1}($$

```
[Out] (-3*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(8*a) + ArcSinh[a*x]^(3/2)/(4*a
^2) + (x^2*ArcSinh[a*x]^(3/2))/2 - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(64*a^2) + (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a^2)
```

Rubi [A] time = 0.219252, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5663, 5758, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}-\frac{3x\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{8a}+\frac{\sinh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\sinh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(8*a) + ArcSinh[a*x]^(3/2)/(4*a
^2) + (x^2*ArcSinh[a*x]^(3/2))/2 - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(64*a^2) + (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a^2)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh^{-1}(ax)^{3/2}\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{1}{4}(3a)\int \frac{x^2\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3}{16}\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{3\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{8a} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x\right.}{16a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh\right.}{16a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh\right.}{32a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{3\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(\right.}{64a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{3\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh}\right.}{32a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2} +
\end{aligned}
$$

Mathematica [A] time = 0.0214305, size = 52, normalized size = 0.43

$$
\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{5}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \text{Gamma}\left(\frac{5}{2},2\sinh^{-1}(ax)\right)}{16\sqrt{2}a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gam
ma[5/2, 2*ArcSinh[a*x]])/(16*Sqrt[2]*a^2)
```

Maple [A] time = 0.091, size = 102, normalized size = 0.8

$$-\frac{\sqrt{2}}{128\sqrt{\pi}a^2}\left(-32\left(\operatorname{Arcsinh}(ax)\right)^{3/2}\sqrt{2}\sqrt{\pi}x^2a^2+24\sqrt{2}\sqrt{\operatorname{Arcsinh}(ax)}\sqrt{\pi}\sqrt{a^2x^2+1}xa-16\left(\operatorname{Arcsinh}(ax)\right)^{3/2}\sqrt{2}\sqrt{\pi}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^(3/2),x)
```

```
[Out] -1/128*2^(1/2)*(-32*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+24*2^(1/2)*
arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a-16*arcsinh(a*x)^(3/2)*2^(
1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-3*Pi*erfi(2^(1/2)*arcsin
h(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x*asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^(3/2), x)
```

3.84 $\int \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{3\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{2a}+\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+x\sinh^{-1}(ax)^{3/2}$$

```
[Out] (-3*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a) + x*ArcSinh[a*x]^(3/2) + (3
*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*
x]]])/(8*a)
```

Rubi [A] time = 0.118619, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5653, 5717, 5657, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{2a}+\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+x\sinh^{-1}(ax)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a) + x*ArcSinh[a*x]^(3/2) + (3
*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*
x]]])/(8*a)
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sinh^{-1}(ax)^{3/2}\,dx = x\sinh^{-1}(ax)^{3/2} - \frac{1}{2}(3a)\int \frac{x\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3}{4}\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a}$$

$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a} + \frac{3\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\right.}{}$$

$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a} + \frac{3\,\text{Subst}\left(\right.}{}$$

$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\sqrt{\pi}\,\mathbf{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\,\mathbf{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}$$

Mathematica [A] time = 0.0282826, size = 47, normalized size = 0.58

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{5}{2}, -\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - \text{Gamma}\left(\frac{5}{2}, \sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamma
[5/2, ArcSinh[a*x]])/(2*a)
```

Maple [A] time = 0.074, size = 65, normalized size = 0.8

$$\frac{1}{8\sqrt{\pi}a}\left(8\,(\text{Arcsinh}(ax))^{3/2}\sqrt{\pi}xa - 12\sqrt{\text{Arcsinh}(ax)}\sqrt{\pi}\sqrt{a^2x^2+1} + 3\pi\,\text{Erf}\left(\sqrt{\text{Arcsinh}(ax)}\right) + 3\pi\,\text{erfi}\left(\sqrt{\text{Arcsinh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2),x)
```

```
[Out] 1/8*(8*arcsinh(a*x)^(3/2)*Pi^(1/2)*x*a-12*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*
x^2+1)^(1/2)+3*Pi*erf(arcsinh(a*x)^(1/2))+3*Pi*erfi(arcsinh(a*x)^(1/2)))/Pi
^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2), x)
```

3.85 $\int \frac{\sinh^{-1}(ax)^{3/2}}{x} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sinh^{-1}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable[ArcSinh[a*x]^(3/2)/x, x]
```

Rubi [A] time = 0.0149503, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{x} dx = \int \frac{\sinh^{-1}(ax)^{3/2}}{x} dx$$

Mathematica [A] time = 0.315237, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/x,x]
```

```
[Out] Integrate[ArcSinh[a*x]^(3/2)/x, x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\mathrm{Arcsinh}\,(ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/x,x)
```

```
[Out] int(arcsinh(a*x)^(3/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/x,x)
```

```
[Out] Integral(asinh(a*x)**(3/2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/x, x)
```

3.86 $\int x^4 \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=379

$$\frac{15\sqrt{\pi}\mathbf{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{128a^5}-\frac{\sqrt{3\pi}\mathbf{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{1280a^5}-\frac{\sqrt{\frac{\pi}{3}}\mathbf{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{240a^5}+\frac{3\sqrt{\frac{\pi}{5}}\mathbf{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{6400a^5}-$$

```
[Out] (2*x*Sqrt[ArcSinh[a*x]])/(5*a^4) - (x^3*Sqrt[ArcSinh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcSinh[a*x]])/100 - (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a
^5) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 +
 a^2*x^2]*ArcSinh[a*x]^(3/2))/(10*a) + (x^5*ArcSinh[a*x]^(5/2))/5 + (15*Sqr
t[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(128*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[Arc
Sinh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1280
*a^5) + (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5) - (15*Sqr
t[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(128*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[A
rcSinh[a*x]]])/(240*a^5) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1
280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5)
```

Rubi [A] time = 0.996392, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$\frac{15\sqrt{\pi}\mathbf{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{128a^5}-\frac{\sqrt{3\pi}\mathbf{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{1280a^5}-\frac{\sqrt{\frac{\pi}{3}}\mathbf{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{240a^5}+\frac{3\sqrt{\frac{\pi}{5}}\mathbf{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{6400a^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^(5/2),x]
```


```
[Out] (2*x*Sqrt[ArcSinh[a*x]])/(5*a^4) - (x^3*Sqrt[ArcSinh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcSinh[a*x]])/100 - (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a
^5) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 +
 a^2*x^2]*ArcSinh[a*x]^(3/2))/(10*a) + (x^5*ArcSinh[a*x]^(5/2))/5 + (15*Sqr
t[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(128*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[Arc
Sinh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1280
*a^5) + (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5) - (15*Sqr
t[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(128*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[A
rcSinh[a*x]]])/(240*a^5) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1
280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^{5/2} - \frac{1}{2}a\int \frac{x^5\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\sinh^{-1}(ax)^{5/2} + \frac{3}{20}\int x^4\sqrt{\sinh^{-1}(ax)}\,dx + \frac{2\int \frac{x^3\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}}{5a}$$

$$= \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} + \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\sinh^{-1}(a$$

$$= -\frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{15a^3}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+}}{}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+}}{}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+}}{}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+}}{}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+}}{}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+}}{}$$

Mathematica [A] time = 0.109468, size = 152, normalized size = 0.4

$$\frac{\frac{27\sqrt{5}\sqrt{\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2},-5\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \frac{625\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{33750\sqrt{\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2},-\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} - 33750}{540000a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((27*Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -5*ArcSinh[a*x]])/Sqrt[-ArcSinh[
a*x]] + (625*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]])/Sqrt[
ArcSinh[a*x]] + (33750*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[-
ArcSinh[a*x]] - 33750*Gamma[7/2, ArcSinh[a*x]] + 625*Sqrt[3]*Gamma[7/2, 3*A
rcSinh[a*x]] - 27*Sqrt[5]*Gamma[7/2, 5*ArcSinh[a*x]])/(540000*a^5)
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int x^4 (\text{Arcsinh}(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(5/2), x)
```

3.87 $\int x^3 \sinh^{-1}(ax)^{5/2}\, dx$

Optimal. Leaf size=247

$$-\frac{15\sqrt{\pi}\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(a\right.}}{512a^4}$$

```
[Out] (-225*Sqrt[ArcSinh[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcSinh[a*x]])/(256*a^2
) + (15*x^4*Sqrt[ArcSinh[a*x]])/256 + (15*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
(3/2))/(64*a^3) - (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(32*a) - (3*
ArcSinh[a*x]^(5/2))/(32*a^4) + (x^4*ArcSinh[a*x]^(5/2))/4 - (15*Sqrt[Pi]*Er
f[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcS
inh[a*x]]])/(512*a^4) - (15*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4
) + (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(512*a^4)
```

Rubi [A] time = 0.71077, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(a\right.}}{512a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^(5/2),x]
```


```
[Out] (-225*Sqrt[ArcSinh[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcSinh[a*x]])/(256*a^2
) + (15*x^4*Sqrt[ArcSinh[a*x]])/256 + (15*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
(3/2))/(64*a^3) - (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(32*a) - (3*
ArcSinh[a*x]^(5/2))/(32*a^4) + (x^4*ArcSinh[a*x]^(5/2))/4 - (15*Sqrt[Pi]*Er
f[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcS
inh[a*x]]])/(512*a^4) - (15*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4
) + (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(512*a^4)
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
```

```
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x^4\sinh^{-1}(ax)^{5/2} - \frac{1}{8}(5a)\int \frac{x^4\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{5x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4\sinh^{-1}(ax)^{5/2} + \frac{15}{64}\int x^3\sqrt{\sinh^{-1}(ax)}\,dx + \frac{15\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}}{32a}$$

$$= \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4\sinh^{-1}($$

$$= -\frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\sinh^-}{32a}$$

$$= -\frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\sinh^-}{32a}$$

$$= -\frac{45\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} - 5$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} -$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} -$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} -$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{64a^3} -$$

Mathematica [A] time = 0.03659, size = 101, normalized size = 0.41

$$\frac{\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-4\sinh^{-1}(ax)\right) - 16\sqrt{2}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-2\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(\text{Gamma}\left(\right.\right.}{2048a^4\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 16*Sqrt[2]*Sqrt[ArcSinh[a
*x]]*Gamma[7/2, -2*ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(-16*Sqrt[2]*Gamma[7
/2, 2*ArcSinh[a*x]] + Gamma[7/2, 4*ArcSinh[a*x]]))/(2048*a^4*Sqrt[-ArcSinh[
a*x]])
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x^3 (\text{Arcsinh}(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(5/2), x)
```

3.88 $\int x^2 \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=210

$$-\frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}+\frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{576a^3}+\frac{15\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{576a^3}$$

```
[Out] (-5*x*Sqrt[ArcSinh[a*x]])/(6*a^2) + (5*x^3*Sqrt[ArcSinh[a*x]])/36 + (5*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x]^(3/2))/(18*a) + (x^3*ArcSinh[a*x]^(5/2))/3 - (15*Sqrt[Pi]*Erf[Sqrt[A
rcSinh[a*x]]])/(64*a^3) + (5*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5
76*a^3) + (15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(64*a^3) - (5*Sqrt[Pi/3]*E
rfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(576*a^3)
```

Rubi [A] time = 0.55076, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}+\frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{576a^3}+\frac{15\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{576a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^(5/2),x]
```


```
[Out] (-5*x*Sqrt[ArcSinh[a*x]])/(6*a^2) + (5*x^3*Sqrt[ArcSinh[a*x]])/36 + (5*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x]^(3/2))/(18*a) + (x^3*ArcSinh[a*x]^(5/2))/3 - (15*Sqrt[Pi]*Erf[Sqrt[A
rcSinh[a*x]]])/(64*a^3) + (5*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5
76*a^3) + (15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(64*a^3) - (5*Sqrt[Pi/3]*E
rfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(576*a^3)
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^{5/2} - \frac{1}{6}(5a)\int \frac{x^3\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{5/2} + \frac{5}{12}\int x^2\sqrt{\sinh^{-1}(ax)}\,dx + \frac{5\int \frac{x\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}}{9a}$$

$$= \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{5}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3}}{18a}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3}}{18a}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3}}{18a}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3}}{18a}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3}}{18a}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3}}{18a}$$

Mathematica [A] time = 0.0364341, size = 101, normalized size = 0.48

$$\frac{\sqrt{3}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2}, -3\sinh^{-1}(ax)\right) - 81\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2}, -\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(81\text{Gamma}\left(\right.\right.}{648a^3\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]] - 81*Sqrt[ArcSinh[a
*x]]*Gamma[7/2, -ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(81*Gamma[7/2, ArcSinh
[a*x]] - Sqrt[3]*Gamma[7/2, 3*ArcSinh[a*x]]))/(648*a^3*Sqrt[-ArcSinh[a*x]])
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int x^2 (\mathrm{Arcsinh}\,(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{arsinh}\,(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(ax)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^(5/2), x)
```

3.89 $\int x\sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=152

$$-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{5x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}{8a}+\frac{\sinh^{-1}(ax)^{5/2}}{4a^2}+\frac{15\sqrt{\sin}}{64}$$

```
[Out] (15*Sqrt[ArcSinh[a*x]])/(64*a^2) + (15*x^2*Sqrt[ArcSinh[a*x]])/32 - (5*x*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(8*a) + ArcSinh[a*x]^(5/2)/(4*a^2) + (x
^2*ArcSinh[a*x]^(5/2))/2 - (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/
(256*a^2) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a^2)
```

Rubi [A] time = 0.336808, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{5x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}{8a}+\frac{\sinh^{-1}(ax)^{5/2}}{4a^2}+\frac{15\sqrt{\sin}}{64}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[ArcSinh[a*x]])/(64*a^2) + (15*x^2*Sqrt[ArcSinh[a*x]])/32 - (5*x*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(8*a) + ArcSinh[a*x]^(5/2)/(4*a^2) + (x
^2*ArcSinh[a*x]^(5/2))/2 - (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/
(256*a^2) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a^2)
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} - \frac{1}{4}(5a)\int\frac{x^2\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} + \frac{15}{16}\int x\sqrt{\sinh^{-1}(ax)}\,dx + \frac{5\int\frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{8a}$$

$$= \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} - \frac{1}{64}(15a$$

$$= \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} - \frac{15\,\mathrm{Subs}}{}$$

$$= \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} + \frac{15\,\mathrm{Sub}}{}$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\,\mathrm{sinh}$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\,\mathrm{sinh}$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\,\mathrm{sinh}$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\,\mathrm{sinh}$$

Mathematica [A] time = 0.0321814, size = 52, normalized size = 0.34

$$\frac{\frac{\sqrt{\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \mathrm{Gamma}\left(\frac{7}{2},2\sinh^{-1}(ax)\right)}{32\sqrt{2}a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[7/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Gam
ma[7/2, 2*ArcSinh[a*x]])/(32*Sqrt[2]*a^2)
```

Maple [A] time = 0.095, size = 136, normalized size = 0.9

$$-\frac{\sqrt{2}}{512\sqrt{\pi}a^2}\left(-128\left(\mathrm{Arcsinh}\left(ax\right)\right)^{5/2}\sqrt{2}\sqrt{\pi}x^2a^2+160\left(\mathrm{Arcsinh}\left(ax\right)\right)^{3/2}\sqrt{2}\sqrt{\pi}\sqrt{a^2x^2+1}xa-120\sqrt{2}\sqrt{\mathrm{Arcsinh}\left(ax\right)}\sqrt{\pi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^(5/2),x)
```

```
[Out] -1/512*2^(1/2)*(-128*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+160*arcsin
h(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a-120*2^(1/2)*arcsinh(a*x
)^(1/2)*Pi^(1/2)*x^2*a^2-64*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)-60*2^(1/2)*
arcsinh(a*x)^(1/2)*Pi^(1/2)+15*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))+15*Pi*erf
i(2^(1/2)*arcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^(5/2), x)
```

3.90 $\int \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=94

$$-\frac{5\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}{2a}+\frac{15\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}+x\sinh^{-1}(ax)^{5/2}+\frac{15}{4}x\sqrt{\sinh^{-1}(ax)}$$

```
[Out] (15*x*Sqrt[ArcSinh[a*x]])/4 - (5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(2*a
) + x*ArcSinh[a*x]^(5/2) + (15*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a) - (
15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a)
```

Rubi [A] time = 0.184399, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$-\frac{5\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}{2a}+\frac{15\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}+x\sinh^{-1}(ax)^{5/2}+\frac{15}{4}x\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(5/2),x]
```


```
[Out] (15*x*Sqrt[ArcSinh[a*x]])/4 - (5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(2*a
) + x*ArcSinh[a*x]^(5/2) + (15*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a) - (
15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a)
```


Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sinh^{-1}(ax)^{5/2}\,dx = x\sinh^{-1}(ax)^{5/2} - \frac{1}{2}(5a)\int \frac{x\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15}{4}\int \sqrt{\sinh^{-1}(ax)}\,dx$$
$$= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} - \frac{1}{8}(15a)\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax}}$$
$$= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} - \frac{15\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(\right.}{8a}$$
$$= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax\right.}{16a}$$
$$= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(}\right.}{8a}$$
$$= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15\sqrt{\pi}\,\mathbf{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a} - \frac{15\sqrt{}}{}$$

Mathematica [A] time = 0.0476373, size = 45, normalized size = 0.48

$$-\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2}, -\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \mathrm{Gamma}\left(\frac{7}{2}, \sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(5/2),x]
```

```
[Out] -((Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gamm
a[7/2, ArcSinh[a*x]])/(2*a)
```

Maple [A] time = 0.072, size = 78, normalized size = 0.8

$$\frac{1}{16\sqrt{\pi}a}\left(16\,(\mathrm{Arcsinh}\,(ax))^{5/2}\sqrt{\pi}xa - 40\,(\mathrm{Arcsinh}\,(ax))^{3/2}\sqrt{\pi}\sqrt{a^2x^2+1} + 60\sqrt{\mathrm{Arcsinh}\,(ax)}\sqrt{\pi}xa + 15\,\pi\,\mathrm{Erf}\left(\sqrt{\mathrm{Arcs}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2),x)
```

```
[Out] 1/16*(16*arcsinh(a*x)^(5/2)*Pi^(1/2)*x*a-40*arcsinh(a*x)^(3/2)*Pi^(1/2)*(a^
2*x^2+1)^(1/2)+60*arcsinh(a*x)^(1/2)*Pi^(1/2)*x*a+15*Pi*erf(arcsinh(a*x)^(1
/2))-15*Pi*erfi(arcsinh(a*x)^(1/2)))/Pi^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2), x)
```

3.91 $\int \frac{\sinh^{-1}(ax)^{5/2}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sinh^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[ArcSinh[a*x]^(5/2)/x, x]
```

Rubi [A] time = 0.0128579, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(5/2)/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^(5/2)/x, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{x} \, dx = \int \frac{\sinh^{-1}(ax)^{5/2}}{x} \, dx$$

Mathematica [A] time = 0.313721, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/x,x]
```

```
[Out] Integrate[ArcSinh[a*x]^(5/2)/x, x]
```

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\mathrm{Arcsinh}\,(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/x,x)
```

```
[Out] int(arcsinh(a*x)^(5/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/x, x)
```

3.92 $\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{3\pi}\,\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{5}}\,\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\sqrt{3\pi}\,\text{Erf}$$

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(32
*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erfi[Sqr
t[3]*Sqrt[ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[
a*x]]])/(32*a^5)
```

Rubi [A] time = 0.204189, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{3\pi}\,\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{5}}\,\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\sqrt{3\pi}\,\text{Erf}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(32
*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erfi[Sqr
t[3]*Sqrt[ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[
a*x]]])/(32*a^5)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{x}} - \frac{3\cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right) dx, x, \sinh^{-1}(ax)\right)}{a^5}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx\right.}{16a^5}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(a\right.}{16a^5}$$

$$= \frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\right.}{8a^5}$$

$$= \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sin\right.}{16a^5}$$

Mathematica [A] time = 0.102216, size = 151, normalized size = 0.93

$$\frac{\frac{\sqrt{5}\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-5\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{5\sqrt{3}\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \frac{10\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - 10\text{Gamm}}{160a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (5*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSi
nh[a*x]] + (10*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[
a*x]] - 10*Gamma[1/2, ArcSinh[a*x]] + 5*Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]]
- Sqrt[5]*Gamma[1/2, 5*ArcSinh[a*x]])/(160*a^5)
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt{\text{Arcsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(arcsinh(a*x)), x)
```

3.93 $\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}$$

```
[Out] -(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(32*a^4)
 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(8*a^4)
```

Rubi [A] time = 0.147836, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(32*a^4)
 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(8*a^4)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh\right.}{8a^4}$$

$$= -\frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{-2x^2}\,dx,\right.}{4a^4}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\right.}{8}$$

<u>Mathematica</u> [A] time = 0.0761502, size = 99, normalized size = 0.91

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -2\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} - 2\sqrt{2}\,\text{Gamma}\left(\frac{1}{2}, 2\sinh^{-1}(ax)\right) + \text{Gamma}\left(\frac{1}{2},\right.}{32a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + (2*
Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]]
 - 2*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] + Gamma[1/2, 4*ArcSinh[a*x]])/(32*a
^4)
```

<u>Maple</u> [F] time = 0.106, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{\text{Arcsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(arcsinh(a*x)), x)
```

3.94 $\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(8*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sq
rt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^3)
```

Rubi [A] time = 0.142904, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5669, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(8*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sq
rt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^3)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax\right.}{8a^3}$$

$$= \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^3} - \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^3} - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{s}\right.}{4a^3}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{si\right.}{8a^3}$$

Mathematica [A] time = 0.0716465, size = 99, normalized size = 0.94

$$\frac{\frac{\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{3\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + 3\text{Gamma}\left(\frac{1}{2},\sinh^{-1}(ax)\right) - \sqrt{3}\text{Gamma}\left(\frac{1}{2},3\sin\right.}{24a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (3*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] +
 3*Gamma[1/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]])/(24*a^3)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{\text{Arcsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(asinh(a*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(arcsinh(a*x)), x)
```

3.95 $\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}$$

```
[Out] -(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2) + (Sqrt[Pi/2]*Erfi[Sq
rt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2)
```

Rubi [A] time = 0.0765867, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2) + (Sqrt[Pi/2]*Erfi[Sq
rt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^2} \\
&= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^2} \\
&= -\frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^2} + \frac{\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^2} \\
&= -\frac{\sqrt{\frac{\pi}{2}}\,\mathbf{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}}\,\mathbf{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0255968, size = 52, normalized size = 0.83

$$
\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -2\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \text{Gamma}\left(\frac{1}{2}, 2\sinh^{-1}(ax)\right)}{4\sqrt{2}a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gam
ma[1/2, 2*ArcSinh[a*x]])/(4*Sqrt[2]*a^2)
```

Maple [A] time = 0.051, size = 37, normalized size = 0.6

$$
-\frac{\sqrt{\pi}\sqrt{2}}{8\,a^2}\left(\text{Erf}\left(\sqrt{2}\sqrt{\text{Arcsinh}\,(ax)}\right) - \text{erfi}\left(\sqrt{2}\sqrt{\text{Arcsinh}\,(ax)}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(1/2),x)
```

```
[Out] -1/8*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arcsinh(a*x)^(1/2))-erfi(2^(1/2)*arcsinh
(a*x)^(1/2)))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(asinh(a*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(arcsinh(a*x)), x)
```

3.96 $\int \frac{1}{\sqrt{\sinh^{-1}(ax)}} dx$

Optimal. Leaf size=43

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}$$

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(2*a) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]
]])/(2*a)
```

Rubi [A] time = 0.0471502, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5657, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(2*a) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]
]])/(2*a)
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a}\\
&= \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a}\\
&= \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0272014, size = 47, normalized size = 1.09

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - \text{Gamma}\left(\frac{1}{2},\sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamma
[1/2, ArcSinh[a*x]])/(2*a)
```

Maple [A] time = 0.036, size = 24, normalized size = 0.6

$$\frac{\sqrt{\pi}}{2a}\left(\mathrm{Erf}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)+\mathrm{erfi}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(1/2),x)
```

```
[Out] 1/2*Pi^(1/2)*(erf(arcsinh(a*x)^(1/2))+erfi(arcsinh(a*x)^(1/2)))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\mathrm{arsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(arcsinh(a*x)), x)
```

3.97 $\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[ArcSinh[a*x]]), x]
```

Rubi [A] time = 0.0134009, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx = \int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.295406, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[ArcSinh[a*x]]), x]
```

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{\mathrm{Arcsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(1/x/arcsinh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\mathrm{arsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(asinh(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

3.98 $\int \frac{1}{x^2\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Rubi [A] time = 0.0154198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\sinh^{-1}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.57002, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Maple [A] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \frac{1}{\sqrt{\operatorname{Arcsinh}(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{\operatorname{arsinh}(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(asinh(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

3.99 $\int \frac{x^4}{\sinh^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=188

$$-\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{5\pi}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}-3\sqrt{3}$$

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(16*a^
5) - (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5) + (Sqrt[Pi]*Erfi
[Sqrt[ArcSinh[a*x]]])/(8*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x
]]])/(16*a^5) + (Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5)
```

Rubi [A] time = 0.185115, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{5\pi}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}-3\sqrt{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^(3/2),x]
```


```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(16*a^
5) - (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5) + (Sqrt[Pi]*Erfi
[Sqrt[ArcSinh[a*x]]])/(8*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x
]]])/(16*a^5) + (Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5)
```


Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{x}} - \frac{9\sinh(3x)}{16\sqrt{x}} + \frac{5\sinh(5x)}{16\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^5} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} + \frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{5\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^5} + \frac{\mathrm{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^5} - \frac{5\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{5\pi}\,\mathrm{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(\cdots\right.}{16a^5}$$

Mathematica [A] time = 0.286323, size = 216, normalized size = 1.15

$$\frac{\sqrt{5}\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -5\sinh^{-1}(ax)\right) - 3\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -3\sinh^{-1}(ax)\right) + 2\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gam}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-E^(-5*ArcSinh[a*x]) + 3/E^(3*ArcSinh[a*x]) - 2/E^ArcSinh[a*x] - 2*E^ArcSi
nh[a*x] + 3*E^(3*ArcSinh[a*x]) - E^(5*ArcSinh[a*x]) + Sqrt[5]*Sqrt[-ArcSinh
[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]] - 3*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/
2, -3*ArcSinh[a*x]] + 2*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] + 2*S
qrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]] - 3*Sqrt[3]*Sqrt[ArcSinh[a*x]]*G
amma[1/2, 3*ArcSinh[a*x]] + Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 5*ArcSinh
[a*x]])/(16*a^5*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int x^4 (\text{Arcsinh}(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\text{arsinh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\text{asinh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**4/asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(3/2), x)
```

3.100 $\int \frac{x^3}{\sinh^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=138

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4} - \frac{2x^3\sqrt{\cdots}}{a\sqrt{\mathrm{si}\cdots}}$$

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[2*Sqrt[Ar
cSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(2*a^4
) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(2*a^4)
```

Rubi [A] time = 0.132808, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4} - \frac{2x^3\sqrt{\cdots}}{a\sqrt{\mathrm{si}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[2*Sqrt[Ar
cSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(2*a^4
) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(2*a^4)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{2\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} + \frac{\mathrm{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^4} - \frac{\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^4} - \frac{\mathrm{Subst}}{}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\mathrm{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a^4} - \frac{\mathrm{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a^4} - $$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}(a}\right.}{4a^4}$$

Mathematica [A] time = 0.0403979, size = 126, normalized size = 0.91

$$\frac{\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -4\sinh^{-1}(ax)\right) - \sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -2\sinh^{-1}(ax)\right) + \sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\mathrm{Gamma}}{4a^4\sqrt{\sinh^{-1}(a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[2]*Sqrt[-ArcSinh[a*
x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*A
rcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + 2*Sinh[2*Arc
Sinh[a*x]] - Sinh[4*ArcSinh[a*x]])/(4*a^4*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^3 \left(\mathrm{Arcsinh}\,(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(3/2), x)
```

3.101 $\int \frac{x^2}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\text{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{2x^2\sqrt{a\cdots}}{a\sqrt{\sinh\cdots}}$$

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(4*a^3) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(4*a^3)
- (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sq
rt[ArcSinh[a*x]]])/(4*a^3)
```

Rubi [A] time = 0.122016, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5665, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\text{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{2x^2\sqrt{a\cdots}}{a\sqrt{\sinh\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(4*a^3) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(4*a^3)
- (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sq
rt[ArcSinh[a*x]]])/(4*a^3)
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int \left(-\frac{\sinh(x)}{4\sqrt{x}} + \frac{3\sinh(3x)}{4\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^3} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} - \frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} - \frac{3\,\mathrm{Subst}}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^3} - \frac{\mathrm{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^3} - \frac{3\,\mathrm{S}}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\mathbf{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3} - \frac{\sqrt{3\pi}\,\mathbf{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3} - \frac{\sqrt{\pi}\,\mathbf{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right.}{4a^3}$$

Mathematica [A] time = 0.13127, size = 140, normalized size = 1.08

$$\frac{\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)-\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)-\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},\text{s}\right.}{4a^3\sqrt{\sinh^{-1}(a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-E^(-3*ArcSinh[a*x]) + E^(-ArcSinh[a*x]) + E^ArcSinh[a*x] - E^(3*ArcSinh[a
*x]) + Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]] - Sqrt[-ArcS
inh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh
[a*x]] + Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 3*ArcSinh[a*x]])/(4*a^3*Sqrt
[ArcSinh[a*x]])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int x^2 (\text{Arcsinh}\,(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(3/2), x)
```

3.102 $\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/a^2
```

Rubi [A] time = 0.0663937, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/a^2
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx &= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a^2}+\frac{\mathrm{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{2\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{2\,\mathrm{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\sinh^{-1}(ax)}\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\sqrt{\frac{\pi}{2}}\,\mathbf{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\mathbf{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0264979, size = 78, normalized size = 0.93

$$\frac{\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{2}a^2\sqrt{\sinh^{-1}(ax)}}-\frac{\text{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)}{\sqrt{2}a^2}-\frac{\sinh\left(2\sinh^{-1}(ax)\right)}{a^2\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/(Sqrt[2]*a^2*Sqrt[ArcSinh
[a*x]]) - Gamma[1/2, 2*ArcSinh[a*x]]/(Sqrt[2]*a^2) - Sinh[2*ArcSinh[a*x]]/(
a^2*Sqrt[ArcSinh[a*x]])
```

Maple [A] time = 0.086, size = 80, normalized size = 1.

$$\frac{\sqrt{2}}{2\sqrt{\pi}a^2\text{Arcsinh}\,(ax)}\left(-2\sqrt{2}\sqrt{\text{Arcsinh}\,(ax)}\sqrt{\pi}\sqrt{a^2x^2+1}xa+\text{Arcsinh}\,(ax)\,\pi\,\text{Erf}\left(\sqrt{2}\sqrt{\text{Arcsinh}\,(ax)}\right)+\text{Arcsinh}\,(ax)\,\pi$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(3/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*2^(1/2)*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a+a
rcsinh(a*x)*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))+arcsinh(a*x)*Pi*erfi(2^(1/2)
*arcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2/arcsinh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(3/2), x)
```

3.103 $\int \frac{1}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[
a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/a
```

Rubi [A] time = 0.1103, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[
a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/a
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + (2a)\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$

$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a} + \frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$

$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{2\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a} + \frac{2\,\mathrm{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a}$$

$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\mathbf{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a} + \frac{\sqrt{\pi}\,\mathbf{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}$$

Mathematica [A] time = 0.0680336, size = 69, normalized size = 1.08

$$\frac{\sqrt{-\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -\sinh^{-1}(ax)\right) + \sqrt{\sinh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, \sinh^{-1}(ax)\right) - e^{-\sinh^{-1}(ax)} - e^{\sinh^{-1}(ax)}}{a\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(-3/2),x]
```

```
[Out] (-E^(-ArcSinh[a*x]) - E^ArcSinh[a*x] + Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcS
inh[a*x]] + Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]])/(a*Sqrt[ArcSinh[a*
x]])
```

Maple [A] time = 0.073, size = 65, normalized size = 1.

$$-\frac{1}{\sqrt{\pi}a\mathrm{Arcsinh}\,(ax)}\left(\mathrm{Arcsinh}\,(ax)\,\pi\,\mathrm{Erf}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right) - \mathrm{Arcsinh}\,(ax)\,\pi\,\mathrm{erfi}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right) + 2\sqrt{\mathrm{Arcsinh}\,(ax)}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(3/2),x)
```

```
[Out] -(arcsinh(a*x)*Pi*erf(arcsinh(a*x)^(1/2))-arcsinh(a*x)*Pi*erfi(arcsinh(a*x)
^(1/2))+2*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh
(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-3/2), x)
```

3.104 $\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0131123, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 0.38304, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]^(3/2)), x]
```

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\text{Arcsinh}\,(ax))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(1/x/arcsinh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{asinh}^{\frac{3}{2}}\,(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*asinh(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

3.105 $\int \frac{x^4}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{5\sqrt{5\pi}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{24a^5}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}}{}$$

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[
ArcSinh[a*x]]) - (20*x^5)/(3*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcSi
nh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^5
) + (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5) + (Sqrt[Pi]*Erf
i[Sqrt[ArcSinh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a
*x]]])/(8*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5)
```

Rubi [A] time = 0.583792, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{5\sqrt{5\pi}\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{24a^5}+\frac{\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^(5/2),x]
```


```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[
ArcSinh[a*x]]) - (20*x^5)/(3*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcSi
nh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^5
) + (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5) + (Sqrt[Pi]*Erf
i[Sqrt[ArcSinh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a
*x]]])/(8*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5)
```


Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(10a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{100}{3}\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{16\int \frac{x^2}{\sqrt{\sinh^{-1}(}}}{a^2}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(\right.}{a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right) dx, x,\right.}{a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{12a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{24a^5} + \frac{25}{}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{12a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\right.}{8a}$$

Mathematica [A] time = 0.333043, size = 343, normalized size = 1.54

$$-\frac{10\sqrt{5}\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-5\sinh^{-1}(ax)\right)+e^{5\sinh^{-1}(ax)}\left(10\sinh^{-1}(ax)+1\right)}{48\sinh^{-1}(ax)^{3/2}} + \frac{6\sqrt{3}\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)+e^{3\sinh^{-1}(ax)}\left(6\sinh^{-1}\right.}{16\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-(E^(5*ArcSinh[a*x])*(1 + 10*ArcSinh[a*x]) + 10*Sqrt[5]*(-ArcSinh[a*x])^(3
/2)*Gamma[1/2, -5*ArcSinh[a*x]])/(48*ArcSinh[a*x]^(3/2)) + (E^(3*ArcSinh[a*
x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -3*Ar
cSinh[a*x]])/(16*ArcSinh[a*x]^(3/2)) - (E^ArcSinh[a*x]*(1 + 2*ArcSinh[a*x])
 + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/(24*ArcSinh[a*x]^(3/2
)) - (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, A
rcSinh[a*x]])/(24*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)) + (1 - 6*ArcSinh[a*x]
+ 6*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 3*ArcSinh[a*x]
])/(16*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)) - (1 - 10*ArcSinh[a*x] + 10*S
qrt[5]*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 5*ArcSinh[a*x]])/(4
8*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)))/a^5
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int x^4 (\mathrm{Arcsinh}\,(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**4/asinh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(5/2), x)
```

3.106 $\int \frac{x^3}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2\sqrt{\pi}\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{\sqrt{2\pi}\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{2\sqrt{\pi}\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}-\frac{\sqrt{2\pi}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}-$$

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[Arc
Sinh[a*x]]) - (16*x^4)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[Pi]*Erf[2*Sqrt[ArcS
inh[a*x]]])/(3*a^4) + (Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
+ (2*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(3*a^4) - (Sqrt[2*Pi]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
```

Rubi [A] time = 0.441375, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5667, 5774, 5669, 5448, 3308, 2180, 2204, 2205, 12}

$$-\frac{2\sqrt{\pi}\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{\sqrt{2\pi}\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{2\sqrt{\pi}\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}-\frac{\sqrt{2\pi}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^(5/2),x]
```


```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[Arc
Sinh[a*x]]) - (16*x^4)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[Pi]*Erf[2*Sqrt[ArcS
inh[a*x]]])/(3*a^4) + (Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
+ (2*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(3*a^4) - (Sqrt[2*Pi]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
```


Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(8a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{64}{3}\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{8\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}}{a^2}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(a\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} +$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a^4} +$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a^4} + \frac{4\,\mathrm{S}}{}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{3a^4} +$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} - \frac{2\sqrt{\pi}\,\mathbf{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4} + \frac{\sqrt{2\pi}\,\mathbf{erf}\left(\sqrt{\right.}}{}$$

Mathematica [A] time = 0.409386, size = 174, normalized size = 1.04

$$\frac{4\sinh^{-1}(ax)\left(-\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)-\sqrt{2}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+e^{-2\sinh^{-1}(ax)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (4*ArcSinh[a*x]*(E^(-2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x]) - Sqrt[2]*Sqrt[-A
rcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma
[1/2, 2*ArcSinh[a*x]]) - 4*ArcSinh[a*x]*(E^(-4*ArcSinh[a*x]) + E^(4*ArcSinh
[a*x]) - 2*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*Sqrt[ArcSinh
[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]) + 2*Sinh[2*ArcSinh[a*x]] - Sinh[4*ArcSin
h[a*x]])/(12*a^4*ArcSinh[a*x]^(3/2))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^3 \left(\text{Arcsinh}\,(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/asinh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(5/2), x)
```

3.107 $\int \frac{x^2}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}-\frac{2x^2\cdots}{3a\,\mathrm{sin}\cdots}$$

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[Arc
Sinh[a*x]]) - (4*x^3)/Sqrt[ArcSinh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]
])/(6*a^3) + (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(2*a^3) - (Sqrt[P
i]*Erfi[Sqrt[ArcSinh[a*x]]])/(6*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSin
h[a*x]]])/(2*a^3)
```

Rubi [A] time = 0.372393, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205, 5657}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}-\frac{2x^2\cdots}{3a\,\mathrm{sin}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[Arc
Sinh[a*x]]) - (4*x^3)/Sqrt[ArcSinh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]
])/(6*a^3) + (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(2*a^3) - (Sqrt[P
i]*Erfi[Sqrt[ArcSinh[a*x]]])/(6*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSin
h[a*x]]])/(2*a^3)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{3a} + (2a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + 12\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{8\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,d}{3a^2}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a^3} + \frac{12}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a^3} + \frac{4\,\mathrm{Su}}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{3a^3} + \frac{8}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a^3} + \frac{4\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\mathrm{si}}\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a^3} + \frac{4\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\mathrm{si}}\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\mathrm{s}}\right.}{2a^3}$$

Mathematica [A] time = 0.138446, size = 225, normalized size = 1.4

$$-\frac{\frac{6\sqrt{3}\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)+e^{3\sinh^{-1}(ax)}\left(6\sinh^{-1}(ax)+1\right)}{12\sinh^{-1}(ax)^{3/2}}+\frac{2\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)+e^{\sinh^{-1}(ax)}\left(2\sinh^{-1}(ax)\right.}{12\sinh^{-1}(ax)^{3/2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-(E^(3*ArcSinh[a*x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x])^(3/2
)*Gamma[1/2, -3*ArcSinh[a*x]])/(12*ArcSinh[a*x]^(3/2)) + (E^ArcSinh[a*x]*(1
 + 2*ArcSinh[a*x]) + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/(12
*ArcSinh[a*x]^(3/2)) + (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^
(3/2)*Gamma[1/2, ArcSinh[a*x]])/(12*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)) - (1
 - 6*ArcSinh[a*x] + 6*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1
/2, 3*ArcSinh[a*x]])/(12*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)))/a^3
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int x^2 \left(\text{Arcsinh}\,(ax)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\text{arsinh}\,(ax)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/asinh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(5/2), x)
```

3.108 $\int \frac{x}{\sinh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=118

$$-\frac{2\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcSinh[a
*x]]) - (8*x^2)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[Arc
Sinh[a*x]]])/(3*a^2) + (2*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
^2)
```

Rubi [A] time = 0.222622, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5667, 5774, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5675}

$$-\frac{2\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcSinh[a
*x]]) - (8*x^2)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[Arc
Sinh[a*x]]])/(3*a^2) + (2*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
^2)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)^{5/2}}\,dx &= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}+\frac{2\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{3a}+\frac{1}{3}(4a)\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}+\frac{16}{3}\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}+\frac{16\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\sinh^{-1}\right.}{3a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}+\frac{16\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{3a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}+\frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{3a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}-\frac{4\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{3a^2}+\frac{4}{\ } \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}-\frac{8\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\sinh^{-1}(ax)}\right)}{3a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\mathrm{e}}{\ }
\end{aligned}
$$

Mathematica [A] time = 0.163837, size = 98, normalized size = 0.83

$$-\frac{2\sinh^{-1}(ax)\left(-\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)-\sqrt{2}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+e^{-2\sinh^{-1}(}\right.}{3a^2\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -(2*ArcSinh[a*x]*(E^(-2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x]) - Sqrt[2]*Sqrt[-
ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamm
a[1/2, 2*ArcSinh[a*x]]) + Sinh[2*ArcSinh[a*x]])/(3*a^2*ArcSinh[a*x]^(3/2))
```

Maple [A] time = 0.086, size = 119, normalized size = 1.

$$-\frac{\sqrt{2}}{3\sqrt{\pi}a^2(\text{Arcsinh}(ax))^2}\left(4(\text{Arcsinh}(ax))^{3/2}\sqrt{2}\sqrt{\pi}x^2a^2+\sqrt{2}\sqrt{\text{Arcsinh}(ax)}\sqrt{\pi}\sqrt{a^2x^2+1}xa+2(\text{Arcsinh}(ax))^2\pi\,\text{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(5/2),x)
```

```
[Out] -1/3*2^(1/2)*(4*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+2^(1/2)*arcsinh
(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a+2*arcsinh(a*x)^2*Pi*erf(2^(1/2)*
arcsinh(a*x)^(1/2))-2*arcsinh(a*x)^2*Pi*erfi(2^(1/2)*arcsinh(a*x)^(1/2))+2*
arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\text{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(5/2), x)
```

3.109 $\int \frac{1}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}+\frac{2\sqrt{\pi}\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x)/(3*Sqrt[ArcSinh[a*x
]]) + (2*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(3*a) + (2*Sqrt[Pi]*Erfi[Sqrt[Ar
cSinh[a*x]]])/(3*a)
```

Rubi [A] time = 0.117399, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5655, 5774, 5657, 3307, 2180, 2204, 2205}

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}+\frac{2\sqrt{\pi}\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x)/(3*Sqrt[ArcSinh[a*x
]]) + (2*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(3*a) + (2*Sqrt[Pi]*Erfi[Sqrt[Ar
cSinh[a*x]]])/(3*a)
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
```

```
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{1}{3}(2a)\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{4}{3}\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{4\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a} + \frac{2\,\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sin\right.}{3a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{4\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{3a} + \frac{4\,\mathrm{Subst}\left(\int e^{x^2}\,dx, x,\right.}{3a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{\pi}\,\mathbf{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a} + \frac{2\sqrt{\pi}\,\mathbf{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}$$

Mathematica [A] time = 0.0983834, size = 105, normalized size = 1.25

$$-\frac{e^{-\sinh^{-1}(ax)}\left(2e^{\sinh^{-1}(ax)}\left(-\sinh^{-1}(ax)\right)^{3/2}\mathbf{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)+2e^{\sinh^{-1}(ax)}\sinh^{-1}(ax)^{3/2}\mathbf{Gamma}\left(\frac{1}{2},\sinh^{-1}(ax)\right.\right.}{3a\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(-5/2),x]
```

```
[Out] -(1 + E^(2*ArcSinh[a*x]) - 2*ArcSinh[a*x] + 2*E^(2*ArcSinh[a*x])*ArcSinh[a*
x] + 2*E^ArcSinh[a*x]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]] + 2*E
^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, ArcSinh[a*x]])/(3*a*E^ArcSinh[a
*x]*ArcSinh[a*x]^(3/2))
```

Maple [A] time = 0.076, size = 81, normalized size = 1.

$$\frac{2}{3\sqrt{\pi}a\left(\mathrm{Arcsinh}\,(ax)\right)^2}\left(-2\left(\mathrm{Arcsinh}\,(ax)\right)^{3/2}\sqrt{\pi}xa+\left(\mathrm{Arcsinh}\,(ax)\right)^2\pi\,\mathrm{Erf}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)+\left(\mathrm{Arcsinh}\,(ax)\right)^2\pi\,\mathrm{erfi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(5/2),x)
```

```
[Out] 2/3*(-2*arcsinh(a*x)^(3/2)*Pi^(1/2)*x*a+arcsinh(a*x)^2*Pi*erf(arcsinh(a*x)^
(1/2))+arcsinh(a*x)^2*Pi*erfi(arcsinh(a*x)^(1/2))-arcsinh(a*x)^(1/2)*Pi^(1/
2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-5/2), x)
```

3.110 $\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0135873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$$

Mathematica [A] time = 0.389101, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]^(5/2)), x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(\mathrm{Arcsinh}\left(ax\right)\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(1/x/arcsinh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{arsinh}\left(ax\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{asinh}^{\frac{5}{2}}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(1/(x*asinh(a*x)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

3.111 $\int \frac{x^4}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}+\frac{9\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{20a^5}-\frac{5\sqrt{5\pi}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}-\frac{9\sqrt{3}}{}$$

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcS
inh[a*x]^(3/2)) - (4*x^5)/(3*ArcSinh[a*x]^(3/2)) - (32*x^2*Sqrt[1 + a^2*x^2
])/(5*a^3*Sqrt[ArcSinh[a*x]]) - (40*x^4*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[ArcSin
h[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(30*a^5) + (9*Sqrt[3*Pi]*Erf[
Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) - (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcS
inh[a*x]]])/(12*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(30*a^5) - (9*Sq
rt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) + (5*Sqrt[5*Pi]*Erfi[Sq
rt[5]*Sqrt[ArcSinh[a*x]]])/(12*a^5)
```

Rubi [A] time = 0.543711, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5667, 5774, 5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}+\frac{9\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{20a^5}-\frac{5\sqrt{5\pi}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}-\frac{9\sqrt{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcS
inh[a*x]^(3/2)) - (4*x^5)/(3*ArcSinh[a*x]^(3/2)) - (32*x^2*Sqrt[1 + a^2*x^2
])/(5*a^3*Sqrt[ArcSinh[a*x]]) - (40*x^4*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[ArcSin
h[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(30*a^5) + (9*Sqrt[3*Pi]*Erf[
Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) - (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcS
inh[a*x]]])/(12*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(30*a^5) - (9*Sq
rt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) + (5*Sqrt[5*Pi]*Erfi[Sq
rt[5]*Sqrt[ArcSinh[a*x]]])/(12*a^5)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + (2a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} + \frac{20}{3}\int \frac{x^4}{\sinh^{-1}(ax)^{3/2}}\,dx + \frac{16\int \frac{x^2}{\sinh^{-1}(}}{5a^2}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\sinh^{-1}(ax)}} +$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\sinh^{-1}(ax)}} -$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\sinh^{-1}(ax)}} +$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\sinh^{-1}(ax)}} +$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\sinh^{-1}(ax)}} -$$

Mathematica [A] time = 0.673252, size = 334, normalized size = 1.17

$$100\sqrt{5}\left(-\sinh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\tfrac{1}{2},-5\sinh^{-1}(ax)\right) - 108\sqrt{3}\left(-\sinh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\tfrac{1}{2},-3\sinh^{-1}(ax)\right) + 8\left(-\sinh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 9*E^(3*ArcSinh
[a*x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) - E^(5*ArcSinh[a*x])*(3 + 1
```

```
0*ArcSinh[a*x] + 100*ArcSinh[a*x]^2) + 100*Sqrt[5]*(-ArcSinh[a*x])^(5/2)*Ga
mma[1/2, -5*ArcSinh[a*x]] - 108*Sqrt[3]*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -3
*ArcSinh[a*x]] + 8*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 +
4*ArcSinh[a*x] - 8*ArcSinh[a*x]^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gam
ma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (9*(1 - 2*ArcSinh[a*x] + 12*ArcSinh
[a*x]^2 - 12*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*Gamma[1/2, 3*Arc
Sinh[a*x]]))/E^(3*ArcSinh[a*x]) + (-3 + 10*ArcSinh[a*x] - 100*ArcSinh[a*x]^
2 + 100*Sqrt[5]*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*Gamma[1/2, 5*ArcSinh[
a*x]])/E^(5*ArcSinh[a*x]))/(240*a^5*ArcSinh[a*x]^(5/2))
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int x^{4} \left(\operatorname{Arcsinh}\left(a x\right)\right)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\operatorname{arsinh}\left(a x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(7/2), x)
```

3.112 $\int \frac{x^3}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=229

$$\frac{16\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}$$

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcSin
h[a*x]^(3/2)) - (16*x^4)/(15*ArcSinh[a*x]^(3/2)) - (16*x*Sqrt[1 + a^2*x^2])
/(5*a^3*Sqrt[ArcSinh[a*x]]) - (128*x^3*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSin
h[a*x]]) + (16*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]
*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^4) + (16*Sqrt[Pi]*Erfi[2*Sqrt[ArcSi
nh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a
^4)
```

Rubi [A] time = 0.427714, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5667, 5774, 5665, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^(7/2),x]
```


```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcSin
h[a*x]^(3/2)) - (16*x^4)/(15*ArcSinh[a*x]^(3/2)) - (16*x*Sqrt[1 + a^2*x^2])
/(5*a^3*Sqrt[ArcSinh[a*x]]) - (128*x^3*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSin
h[a*x]]) + (16*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]
*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^4) + (16*Sqrt[Pi]*Erfi[2*Sqrt[ArcSi
nh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a
^4)
```


Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{6\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(8a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} + \frac{64}{15}\int \frac{x^3}{\sinh^{-1}(ax)^{3/2}}\,dx + \frac{8\int \frac{x}{\sinh^{-1}(}}{5a^2}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}}$$

Mathematica [A] time = 0.661123, size = 210, normalized size = 0.92

$$\frac{4\sinh^{-1}(ax)\left(4\sqrt{2}\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right) + 4\sqrt{2}\sinh^{-1}(ax)^{3/2}\text{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right) + e^{-2\mathrm{s}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (4*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x
])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*Arc
```

```
Sinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]) - 4*A
rcSinh[a*x]*((1 - 8*ArcSinh[a*x])/E^(4*ArcSinh[a*x]) + E^(4*ArcSinh[a*x])*(
1 + 8*ArcSinh[a*x]) + 16*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]]
+ 16*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]]) + 6*Sinh[2*ArcSinh[a*x]
] - 3*Sinh[4*ArcSinh[a*x]])/(60*a^4*ArcSinh[a*x]^(5/2))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x^3 (\text{Arcsinh}\,(ax))^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}\,(ax)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(7/2), x)
```

3.113 $\int \frac{x^2}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=222

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}-\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}-\frac{24x^2}{5a\sqrt{}}$$

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (8*x)/(15*a^2*ArcSinh
[a*x]^(3/2)) - (4*x^3)/(5*ArcSinh[a*x]^(3/2)) - (16*Sqrt[1 + a^2*x^2])/(15*
a^3*Sqrt[ArcSinh[a*x]]) - (24*x^2*Sqrt[1 + a^2*x^2])/(5*a*Sqrt[ArcSinh[a*x]
]) + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(15*a^3) - (3*Sqrt[3*Pi]*Erf[Sqrt[3
]*Sqrt[ArcSinh[a*x]]])/(5*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a^
3) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5*a^3)
```

Rubi [A] time = 0.433931, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5667, 5774, 5665, 3308, 2180, 2204, 2205, 5655, 5779}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}-\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}-\frac{24x^2}{5a\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (8*x)/(15*a^2*ArcSinh
[a*x]^(3/2)) - (4*x^3)/(5*ArcSinh[a*x]^(3/2)) - (16*Sqrt[1 + a^2*x^2])/(15*
a^3*Sqrt[ArcSinh[a*x]]) - (24*x^2*Sqrt[1 + a^2*x^2])/(5*a*Sqrt[ArcSinh[a*x]
]) + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(15*a^3) - (3*Sqrt[3*Pi]*Erf[Sqrt[3
]*Sqrt[ArcSinh[a*x]]])/(5*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a^
3) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5*a^3)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{4\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(6a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} + \frac{12}{5}\int \frac{x^2}{\sinh^{-1}(ax)^{3/2}}\,dx + \frac{8\int \frac{1}{\sinh^{-1}(ax}}{15a^2}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\sinh^{-1}(ax)}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\sinh^{-1}(ax)}}$$

Mathematica [A] time = 0.352793, size = 221, normalized size = 1.

$$\frac{36\sqrt{3}\left(-\sinh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)-4\left(-\sinh^{-1}(ax)\right)^{5/2}\text{Gamma}\left(\frac{1}{2},-\sinh^{-1}(ax)\right)+e^{-\sinh^{-1}(ax)}\left(-4e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) - 3*E^(3*ArcSinh[a*
x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) + 36*Sqrt[3]*(-ArcSinh[a*x])^(
5/2)*Gamma[1/2, -3*ArcSinh[a*x]] - 4*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -ArcS
inh[a*x]] + (3 - 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2 - 4*E^ArcSinh[a*x]*ArcSi
nh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (-3 + 6*ArcSinh[a*
x] - 36*ArcSinh[a*x]^2 + 36*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*G
amma[1/2, 3*ArcSinh[a*x]])/E^(3*ArcSinh[a*x]))/(60*a^3*ArcSinh[a*x]^(5/2))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x^{2}\left(\text{Arcsinh}\left(ax\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\text{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(7/2), x)
```

3.114 $\int \frac{x}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{8\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}-\frac{32x\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2x\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4}{15a^2\sinh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - 4/(15*a^2*ArcSinh[a*x]^
(3/2)) - (8*x^2)/(15*ArcSinh[a*x]^(3/2)) - (32*x*Sqrt[1 + a^2*x^2])/(15*a*S
qrt[ArcSinh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2
) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2)
```

Rubi [A] time = 0.211751, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5667, 5774, 5665, 3307, 2180, 2204, 2205, 5675}

$$\frac{8\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}-\frac{32x\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2x\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4}{15a^2\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - 4/(15*a^2*ArcSinh[a*x]^
(3/2)) - (8*x^2)/(15*ArcSinh[a*x]^(3/2)) - (32*x*Sqrt[1 + a^2*x^2])/(15*a*S
qrt[ArcSinh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2
) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2)
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)^{7/2}}\,dx &= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{2\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(4a)\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx\\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} + \frac{16}{15}\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx\\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{32\,\mathrm{Subst}\left(\int \frac{\mathrm{co}\ldots}{\ldots}\right.}{\ldots}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\mathrm{Subst}\left(\int \frac{e^{-\ldots}}{\sqrt{\ldots}}\right.}{\ldots}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{32\,\mathrm{Subst}\left(\int e^{-\ldots}\right.}{\ldots}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{8\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\ldots\right.}{15a\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.295126, size = 118, normalized size = 0.8

$$
-\frac{2\sinh^{-1}(ax)\left(4\sqrt{2}\left(-\sinh^{-1}(ax)\right)^{3/2}\,\mathrm{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+4\sqrt{2}\sinh^{-1}(ax)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+e^{-2\ldots}\right.}{15a^2\sinh^{-1}(ax)^{5/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSinh[a*x]^(7/2),x]
```

```
[Out] -(2*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSinh[a*
x])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*Ar
cSinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]) + 3*
Sinh[2*ArcSinh[a*x]])/(15*a^2*ArcSinh[a*x]^(5/2))
```

Maple [A] time = 0.092, size = 147, normalized size = 1.

$$\frac{\sqrt{2}}{15\sqrt{\pi}\,a^{2}\left(\operatorname{Arcsinh}\left(ax\right)\right)^{3}}\left(-16\left(\operatorname{Arcsinh}\left(ax\right)\right)^{5/2}\sqrt{2}\sqrt{\pi}\sqrt{a^{2}x^{2}+1}\,xa-4\left(\operatorname{Arcsinh}\left(ax\right)\right)^{3/2}\sqrt{2}\sqrt{\pi}\,x^{2}a^{2}-3\sqrt{2}\sqrt{\operatorname{Arcsinh}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(7/2),x)
```

```
[Out] 1/15*2^(1/2)*(-16*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a
-4*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2-3*2^(1/2)*arcsinh(a*x)^(1/2)
*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a+8*arcsinh(a*x)^3*Pi*erf(2^(1/2)*arcsinh(a*x
)^(1/2))+8*arcsinh(a*x)^3*Pi*erfi(2^(1/2)*arcsinh(a*x)^(1/2))-2*arcsinh(a*x
)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(7/2), x)
```

3.115 $\int \frac{1}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{8\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}-\frac{4x}{15\sinh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x)/(15*ArcSinh[a*x]^(3
/2)) - (8*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSinh[a*x]]) - (4*Sqrt[Pi]*Erf[Sq
rt[ArcSinh[a*x]]])/(15*a) + (4*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a)
```

Rubi [A] time = 0.189101, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}-\frac{4x}{15\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x)/(15*ArcSinh[a*x]^(3
/2)) - (8*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSinh[a*x]]) - (4*Sqrt[Pi]*Erf[Sq
rt[ArcSinh[a*x]]])/(15*a) + (4*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a)
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
```

```
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{1}{5}(2a)\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} + \frac{4}{15}\int \frac{1}{\sinh^{-1}(ax)^{3/2}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{1}{15}(8a)\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{15a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{15a} + \frac{4\ldots}{}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{15a} + \ldots$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} - \frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\ldots}\right.}{1\ldots}$$

Mathematica [A] time = 0.18052, size = 111, normalized size = 0.99

$$\frac{8\left(-\sinh^{-1}(ax)\right)^{5/2}\mathrm{Gamma}\left(\frac{1}{2}, -\sinh^{-1}(ax)\right) + e^{-\sinh^{-1}(ax)}\left(8e^{\sinh^{-1}(ax)}\sinh^{-1}(ax)^{5/2}\mathrm{Gamma}\left(\frac{1}{2}, \sinh^{-1}(ax)\right) - 8\sinh^{-}\ldots\right.}{30a\sinh^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(-7/2),x]
```

```
[Out] (-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 8*(-ArcSinh[a*
x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 + 4*ArcSinh[a*x] - 8*ArcSinh[a*x]
^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSin
h[a*x])/(30*a*ArcSinh[a*x]^(5/2))
```

Maple [A] time = 0.082, size = 105, normalized size = 0.9

$$-\frac{2}{15\sqrt{\pi}\,a\,(\mathrm{Arcsinh}\,(ax))^3}\left(2\,(\mathrm{Arcsinh}\,(ax))^3\,\pi\,\mathrm{Erf}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)-2\,(\mathrm{Arcsinh}\,(ax))^3\,\pi\,\mathrm{erfi}\left(\sqrt{\mathrm{Arcsinh}\,(ax)}\right)+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(7/2),x)
```

```
[Out] -2/15*(2*arcsinh(a*x)^3*Pi*erf(arcsinh(a*x)^(1/2))-2*arcsinh(a*x)^3*Pi*erfi
(arcsinh(a*x)^(1/2))+4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(5/2)*Pi^(1/2)+2*arcs
inh(a*x)^(3/2)*Pi^(1/2)*x*a+3*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)
)/Pi^(1/2)/a/arcsinh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\mathrm{arsinh}\,(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-7/2), x)
```

3.116 $\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(ax)^{7/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a*x]^(7/2)), x]
```

Rubi [A] time = 0.0143787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^(7/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(7/2)), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$$

Mathematica [A] time = 0.391533, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^(7/2)),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a*x]^(7/2)), x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(\mathrm{Arcsinh}\left(ax\right)\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(1/x/arcsinh(a*x)^(7/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

3.117 $\int x^m \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=53

$$\frac{x^{m+1}\sinh^{-1}(ax)^4}{m+1} - \frac{4a\text{Unintegrable}\left(\frac{x^{m+1}\sinh^{-1}(ax)^3}{\sqrt{a^2x^2+1}}, x\right)}{m+1}$$

```
[Out] (x^(1 + m)*ArcSinh[a*x]^4)/(1 + m) - (4*a*Unintegrable[(x^(1 + m)*ArcSinh[a
*x]^3)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi [A] time = 0.110359, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sinh^{-1}(ax)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^4,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^4)/(1 + m) - (4*a*Defer[Int][(x^(1 + m)*ArcSinh[a*x
]^3)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^4 \, dx = \frac{x^{1+m}\sinh^{-1}(ax)^4}{1+m} - \frac{(4a)\int \frac{x^{1+m}\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}} \, dx}{1+m}$$

Mathematica [A] time = 0.825479, size = 0, normalized size = 0.

$$\int x^m \sinh^{-1}(ax)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^4,x]
```

```
[Out] Integrate[x^m*ArcSinh[a*x]^4, x]
```

Maple [A] time = 0.713, size = 0, normalized size = 0.

$$\int x^m (\mathrm{Arcsinh}\,(ax))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^4,x)
```

```
[Out] int(x^m*arcsinh(a*x)^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m\,\mathrm{arsinh}\,(ax)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \,\mathrm{asinh}^4\,(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**4,x)
```

```
[Out] Integral(x**m*asinh(a*x)**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arsinh}(ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^4, x)
```

3.118 $\int x^m \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=53

$$\frac{x^{m+1}\sinh^{-1}(ax)^3}{m+1} - \frac{3a\text{Unintegrable}\left(\frac{x^{m+1}\sinh^{-1}(ax)^2}{\sqrt{a^2x^2+1}}, x\right)}{m+1}$$

```
[Out] (x^(1 + m)*ArcSinh[a*x]^3)/(1 + m) - (3*a*Unintegrable[(x^(1 + m)*ArcSinh[a
*x]^2)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi [A] time = 0.105995, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \sinh^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^3,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^3)/(1 + m) - (3*a*Defer[Int][(x^(1 + m)*ArcSinh[a*x
]^2)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^3 \, dx = \frac{x^{1+m}\sinh^{-1}(ax)^3}{1+m} - \frac{(3a)\int \frac{x^{1+m}\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx}{1+m}$$

Mathematica [A] time = 0.743325, size = 0, normalized size = 0.

$$\int x^m \sinh^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^3,x]
```

```
[Out] Integrate[x^m*ArcSinh[a*x]^3, x]
```

Maple [A] time = 0.547, size = 0, normalized size = 0.

$$\int x^m (\operatorname{Arcsinh}(ax))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^3,x)
```

```
[Out] int(x^m*arcsinh(a*x)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \operatorname{arsinh}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{asinh}^3(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**3,x)
```

```
[Out] Integral(x**m*asinh(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arsinh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^3, x)
```

3.119 $\int x^m \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=137

$$\frac{2a^2x^{m+3}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},-a^2x^2\right)}{m^3+6m^2+11m+6}-\frac{2ax^{m+2}\sinh^{-1}(ax)\text{Hypergeometric2F1}}{m^2+3m+2}$$

```
[Out] (x^(1 + m)*ArcSinh[a*x]^2)/(1 + m) - (2*a*x^(2 + m)*ArcSinh[a*x]*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) + (2*a^2*x^
(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
-(a^2*x^2)])/(6 + 11*m + 6*m^2 + m^3)
```

Rubi [A] time = 0.0998867, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5661, 5762}

$$\frac{2a^2x^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-a^2x^2\right)}{m^3+6m^2+11m+6}-\frac{2ax^{m+2}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\sinh^{-1}(ax)^2}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^2)/(1 + m) - (2*a*x^(2 + m)*ArcSinh[a*x]*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) + (2*a^2*x^
(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
-(a^2*x^2)])/(6 + 11*m + 6*m^2 + m^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
```

```
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^2\,dx = \frac{x^{1+m}\sinh^{-1}(ax)^2}{1+m} - \frac{(2a)\int \frac{x^{1+m}\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{1+m}$$
$$= \frac{x^{1+m}\sinh^{-1}(ax)^2}{1+m} - \frac{2ax^{2+m}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2} + \frac{2a^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\right.}{6+11m+6}$$

Mathematica [A] time = 0.0382418, size = 123, normalized size = 0.9

$$\frac{x^{m+1}\left(2a^2x^2\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},-a^2x^2\right)+(m+3)\sinh^{-1}(ax)\left((m+2)\sinh^{-1}(\right.\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*((3 + m)*ArcSinh[a*x]*((2 + m)*ArcSinh[a*x] - 2*a*x*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]) + 2*a^2*x^2*HypergeometricPFQ
[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, -(a^2*x^2)]))/((1 + m)*(2
 + m)*(3 + m))
```

Maple [F] time = 0.55, size = 0, normalized size = 0.

$$\int x^m (\text{Arcsinh}(ax))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^2,x)
```

```
[Out] int(x^m*arcsinh(a*x)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \operatorname{arsinh}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{asinh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**2,x)
```

```
[Out] Integral(x**m*asinh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arsinh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^2, x)
```

3.120 $\int x^m \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=60

$$\frac{x^{m+1}\sinh^{-1}(ax)}{m+1}-\frac{ax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}$$

```
[Out] (x^(1 + m)*ArcSinh[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 +
 m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rubi [A] time = 0.0195273, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5661, 364}

$$\frac{x^{m+1}\sinh^{-1}(ax)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 +
 m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)\,dx = \frac{x^{1+m}\sinh^{-1}(ax)}{1+m} - \frac{a\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx}{1+m}$$
$$= \frac{x^{1+m}\sinh^{-1}(ax)}{1+m} - \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}$$

Mathematica [A] time = 0.0222289, size = 55, normalized size = 0.92

$$\frac{x^{m+1}\left((m+2)\sinh^{-1}(ax) - ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x],x]
```

```
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x] - a*x*Hypergeometric2F1[1/2, (2 + m)/2, (4
 + m)/2, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F] time = 0.546, size = 0, normalized size = 0.

$$\int x^m \text{Arcsinh}\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x),x)
```

```
[Out] int(x^m*arcsinh(a*x),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{m}\,\text{arsinh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\text{asinh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x),x)
```

```
[Out] Integral(x**m*asinh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\,\text{arsinh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x), x)
```

3.121 $\int \frac{x^m}{\sinh^{-1}(ax)} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x^m}{\sinh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^m/ArcSinh[a*x], x]
```

Rubi [A] time = 0.0148593, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sinh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcSinh[a*x],x]
```

```
[Out] Defer[Int][x^m/ArcSinh[a*x], x]
```

Rubi steps

$$\int \frac{x^m}{\sinh^{-1}(ax)} dx = \int \frac{x^m}{\sinh^{-1}(ax)} dx$$

Mathematica [A] time = 0.396761, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sinh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcSinh[a*x],x]
```

```
[Out] Integrate[x^m/ArcSinh[a*x], x]
```

Maple [A] time = 0.429, size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{Arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x),x)
```

```
[Out] int(x^m/arcsinh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arcsinh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m/arcsinh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x),x)
```

```
[Out] Integral(x**m/asinh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/arcsinh(a*x), x)
```

3.122 $\int \frac{x^m}{\sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x^m}{\sinh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[x^m/ArcSinh[a*x]^2, x]
```

Rubi [A] time = 0.0136401, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sinh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcSinh[a*x]^2,x]
```

```
[Out] Defer[Int][x^m/ArcSinh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m}{\sinh^{-1}(ax)^2} dx = \int \frac{x^m}{\sinh^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.410932, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sinh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcSinh[a*x]^2,x]
```

```
[Out] Integrate[x^m/ArcSinh[a*x]^2, x]
```

Maple [A] time = 0.435, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\mathrm{Arcsinh}\,(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x)^2,x)
```

```
[Out] int(x^m/arcsinh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x^m+\left(a^3x^3+ax\right)x^m}{\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int \frac{\left(a^3(m+1)x^3+a(m-1)x\right)\left(a^2x^2+1\right)x^m+\left(2\,a^4(m+1)x^4+a^2\cdots\right.}{\left(a^5x^5+\left(a^2x^2+1\right)a^3x^3+2\,a^3x^3+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^2*x^2 + 1)^(3/2)*x^m + (a^3*x^3 + a*x)*x^m)/((a^3*x^2 + sqrt(a^2*x^2 +
 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate(((a^3*(m + 1)*x^3
+ a*(m - 1)*x)*(a^2*x^2 + 1)*x^m + (2*a^4*(m + 1)*x^4 + a^2*(3*m + 1)*x^2 +
 m)*sqrt(a^2*x^2 + 1)*x^m + (a^5*(m + 1)*x^5 + 2*a^3*(m + 1)*x^3 + a*(m + 1
)*x)*x^m)/((a^5*x^5 + (a^2*x^2 + 1)*a^3*x^3 + 2*a^3*x^3 + a*x + 2*(a^4*x^4
+ a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\mathrm{arsinh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arcsinh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x)**2,x)
```

```
[Out] Integral(x**m/asinh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arcsinh(a*x)^2, x)
```

3.123 $\int x^m \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \sinh^{-1}(ax)^{5/2}, x\right)$$

```
[Out] Unintegrable[x^m*ArcSinh[a*x]^(5/2), x]
```

Rubi [A] time = 0.0150432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \sinh^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^m*ArcSinh[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^{5/2}\,dx = \int x^m \sinh^{-1}(ax)^{5/2}\,dx$$

Mathematica [A] time = 1.05801, size = 0, normalized size = 0.

$$\int x^m \sinh^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^(5/2),x]
```

```
[Out] Integrate[x^m*ArcSinh[a*x]^(5/2), x]
```

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int x^m (\mathrm{Arcsinh}\,(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arsinh}\,(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.124 $\int x^m \sinh^{-1}(ax)^{3/2}\, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \sinh^{-1}(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable[x^m*ArcSinh[a*x]^(3/2), x]
```

Rubi [A] time = 0.0141627, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sinh^{-1}(ax)^{3/2}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*ArcSinh[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^{3/2}\, dx = \int x^m \sinh^{-1}(ax)^{3/2}\, dx$$

Mathematica [A] time = 1.05965, size = 0, normalized size = 0.

$$\int x^m \sinh^{-1}(ax)^{3/2}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*ArcSinh[a*x]^(3/2), x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int x^m (\mathrm{Arcsinh}\,(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.125 $\int x^m \sqrt{\sinh^{-1}(ax)}\, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x^m \sqrt{\sinh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^m*Sqrt[ArcSinh[a*x]], x]
```

Rubi [A] time = 0.0131903, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \sqrt{\sinh^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Defer[Int][x^m*Sqrt[ArcSinh[a*x]], x]
```

Rubi steps

$$\int x^m \sqrt{\sinh^{-1}(ax)}\, dx = \int x^m \sqrt{\sinh^{-1}(ax)}\, dx$$

Mathematica [A] time = 1.32783, size = 0, normalized size = 0.

$$\int x^m \sqrt{\sinh^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Integrate[x^m*Sqrt[ArcSinh[a*x]], x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int x^m \sqrt{\operatorname{Arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(asinh(a*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.126 $\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x^m/Sqrt[ArcSinh[a*x]], x]
```

Rubi [A] time = 0.013785, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Defer[Int][x^m/Sqrt[ArcSinh[a*x]], x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}} \, dx = \int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.19762, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Integrate[x^m/Sqrt[ArcSinh[a*x]], x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\operatorname{Arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^m/arcsinh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(asinh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(arcsinh(a*x)), x)
```

3.127 $\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x^m}{\sinh^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[x^m/ArcSinh[a*x]^(3/2), x]
```

Rubi [A] time = 0.0148467, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcSinh[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m/ArcSinh[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 1.13339, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcSinh[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m/ArcSinh[a*x]^(3/2), x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{Arcsinh}\left(ax\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^m/arcsinh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**m/asinh(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/arcsinh(a*x)^(3/2), x)
```

3.128 $\int (bx)^m \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left((bx)^m \sinh^{-1}(ax)^n, x\right)$$

```
[Out] Unintegrable[(b*x)^m*ArcSinh[a*x]^n, x]
```

Rubi [A] time = 0.0176584, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (bx)^m \sinh^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m*ArcSinh[a*x]^n,x]
```

```
[Out] Defer[Int][(b*x)^m*ArcSinh[a*x]^n, x]
```

Rubi steps

$$\int (bx)^m \sinh^{-1}(ax)^n \, dx = \int (bx)^m \sinh^{-1}(ax)^n \, dx$$

Mathematica [A] time = 0.737356, size = 0, normalized size = 0.

$$\int (bx)^m \sinh^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m*ArcSinh[a*x]^n,x]
```

```
[Out] Integrate[(b*x)^m*ArcSinh[a*x]^n, x]
```

Maple [A] time = 0.064, size = 0, normalized size = 0.

$$\int (bx)^m (\mathrm{Arcsinh}(ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arcsinh(a*x)^n,x)
```

```
[Out] int((b*x)^m*arcsinh(a*x)^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*arcsinh(a*x)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx)^m \operatorname{arsinh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arcsinh(a*x)^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{asinh}^n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*asinh(a*x)**n,x)
```

```
[Out] Integral((b*x)**m*asinh(a*x)**n, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] Timed out
```

3.129 $\int x^4 \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=173

$$\frac{5^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-5\sinh^{-1}(ax)\right)}{32a^5}-\frac{3^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{32a^5}$$

```
[Out] (5^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(32*a^5*(-ArcSinh
[a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(32*3^n*a^5*(-Ar
cSinh[a*x])^n) + (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(16*a^5*(-Arc
Sinh[a*x])^n) - Gamma[1 + n, ArcSinh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcSin
h[a*x]]/(32*3^n*a^5) - (5^(-1 - n)*Gamma[1 + n, 5*ArcSinh[a*x]])/(32*a^5)
```

Rubi [A] time = 0.216363, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5669, 5448, 3307, 2181}

$$\frac{5^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-5\sinh^{-1}(ax)\right)}{32a^5}-\frac{3^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{32a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^n,x]
```


```
[Out] (5^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(32*a^5*(-ArcSinh
[a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(32*3^n*a^5*(-Ar
cSinh[a*x])^n) + (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(16*a^5*(-Arc
Sinh[a*x])^n) - Gamma[1 + n, ArcSinh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcSin
h[a*x]]/(32*3^n*a^5) - (5^(-1 - n)*Gamma[1 + n, 5*ArcSinh[a*x]])/(32*a^5)
```


Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \sinh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh^4(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{8}x^n\cosh(x) - \frac{3}{16}x^n\cosh(3x) + \frac{1}{16}x^n\cosh(5x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int x^n\cosh(5x)\,dx, x, \sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int x^n\cosh(x)\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Subst}\left(\int \right.}{}\\
&= \frac{\text{Subst}\left(\int e^{-5x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int e^{5x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \text{s}\right.}{16a^5}\\
&= \frac{5^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -5\sinh^{-1}(ax)\right)}{32a^5} - \frac{3^{-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma}{32a^5}
\end{aligned}
$$

Mathematica [A] time = 0.146366, size = 145, normalized size = 0.84

$$
\frac{5^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1, -5\sinh^{-1}(ax)\right) - 5\,3^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^n,x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(5^n*(-ArcSinh[a*x])^n) - (
5*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(3^n*(-ArcSinh[a*x])^n) + (
10*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - 10*Gamma
[1 + n, ArcSinh[a*x]] + (5*Gamma[1 + n, 3*ArcSinh[a*x]])/3^n - Gamma[1 + n,
 5*ArcSinh[a*x]]/5^n)/(160*a^5)
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int x^{4} \left(\mathrm{Arcsinh}\left(ax\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^n,x)
```

```
[Out] int(x^4*arcsinh(a*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{4} \operatorname{arsinh}\left(ax\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{4} \operatorname{arsinh}\left(ax\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsinh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{asinh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**n,x)
```

```
[Out] Integral(x**4*asinh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsinh}(ax)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^n, x)
```

3.130 $\int x^3 \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=119

$$\frac{2^{-2(n+3)} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-4\sinh^{-1}(ax)\right)}{a^4} - \frac{2^{-n-4} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\bigl(}{a^4}$$

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]])/(2^(2*(3 + n))*a^4*(-ArcSinh
[a*x])^n) - (2^(-4 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^4*
(-ArcSinh[a*x])^n) - (2^(-4 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^4 + Gamma[
1 + n, 4*ArcSinh[a*x]]/(2^(2*(3 + n))*a^4)
```

Rubi [A] time = 0.174119, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5669, 5448, 3308, 2181}

$$\frac{2^{-2(n+3)} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-4\sinh^{-1}(ax)\right)}{a^4} - \frac{2^{-n-4} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\bigl(}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^n,x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]])/(2^(2*(3 + n))*a^4*(-ArcSinh
[a*x])^n) - (2^(-4 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^4*
(-ArcSinh[a*x])^n) - (2^(-4 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^4 + Gamma[
1 + n, 4*ArcSinh[a*x]]/(2^(2*(3 + n))*a^4)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sinh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh^3(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}x^n \sinh(2x) + \frac{1}{8}x^n \sinh(4x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int x^n \sinh(4x)\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int x^n \sinh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}\\
&= -\frac{\text{Subst}\left(\int e^{-4x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int e^{4x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x\right.}{8a^4}\\
&= \frac{4^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -4\sinh^{-1}(ax)\right)}{a^4} - \frac{2^{-4-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^{}}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.0653109, size = 99, normalized size = 0.83

$$\frac{4^{-n-3}\left(-\sinh^{-1}(ax)\right)^{-n}\left(\left(-\sinh^{-1}(ax)\right)^n\left(\text{Gamma}\left(n+1, 4\sinh^{-1}(ax)\right) - 2^{n+2}\text{Gamma}\left(n+1, 2\sinh^{-1}(ax)\right)\right) + \sinh^{-1}(\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^n,x]
```

```
[Out] (4^(-3 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]] - 2^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(2^(2 + n)*Ga
mma[1 + n, 2*ArcSinh[a*x]]) + Gamma[1 + n, 4*ArcSinh[a*x]])))/(a^4*(-ArcSin
```

```
h[a*x])^n)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^3 (\text{Arcsinh}\,(ax))^n \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^n,x)
```

```
[Out] int(x^3*arcsinh(a*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}\,(ax)^n \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \operatorname{arsinh}\,(ax)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsinh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{asinh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**n,x)
```

```
[Out] Integral(x**3*asinh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n, x)
```

3.131 $\int x^2 \sinh^{-1}(ax)^n\,dx$

Optimal. Leaf size=113

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^3}-\frac{\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\right.}{8a^3}$$

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^3*(-ArcSinh[
a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^3*(-ArcSinh[a*
x])^n) + Gamma[1 + n, ArcSinh[a*x]]/(8*a^3) - (3^(-1 - n)*Gamma[1 + n, 3*Ar
cSinh[a*x]])/(8*a^3)
```

Rubi [A] time = 0.145062, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5669, 5448, 3307, 2181}

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^3}-\frac{\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^n,x]
```

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^3*(-ArcSinh[
a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^3*(-ArcSinh[a*
x])^n) + Gamma[1 + n, ArcSinh[a*x]]/(8*a^3) - (3^(-1 - n)*Gamma[1 + n, 3*Ar
cSinh[a*x]])/(8*a^3)
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh^2(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}x^n\cosh(x) + \frac{1}{4}x^n\cosh(3x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int x^n \cosh(x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int x^n\cosh(3x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}\\
&= \frac{\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \sin\right.}{8a^3}\\
&= \frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -3\sinh^{-1}(ax)\right)}{8a^3} - \frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1\right.}{8a^3}
\end{aligned}$$

Mathematica [A] time = 0.0702013, size = 97, normalized size = 0.86

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1, -3\sinh^{-1}(ax)\right) - \sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1, -\text{si}\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^n,x]
```

```
[Out] ((3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(-ArcSinh[a*x])^
n - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[
```

```
1 + n, ArcSinh[a*x]] - 3^(-1 - n)*Gamma[1 + n, 3*ArcSinh[a*x]])/(8*a^3)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x^2 (\mathrm{Arcsinh}\,(ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^n,x)
```

```
[Out] int(x^2*arcsinh(a*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \,\mathrm{arsinh}\,(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2 \,\mathrm{arsinh}\,(ax)^n \,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \,\mathrm{asinh}^n\,(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**n,x)
```

```
[Out] Integral(x**2*asinh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n, x)
```

3.132 $\int x \sinh^{-1}(ax)^n\,dx$

Optimal. Leaf size=59

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^2}$$

```
[Out] (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^2*(-ArcSinh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Rubi [A] time = 0.0841649, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5669, 5448, 12, 3308, 2181}

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^2*(-ArcSinh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n\cosh(x)\sinh(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{1}{2}x^n\sinh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int x^n\sinh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}\\
&= -\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^2}\\
&= \frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -2\sinh^{-1}(ax)\right)}{a^2} + \frac{2^{-3-n}\Gamma\left(1+n, 2\sinh^{-1}(ax)\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0149539, size = 59, normalized size = 1.

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1, -2\sinh^{-1}(ax)\right)}{a^2} + \frac{2^{-n-3}\text{Gamma}\left(n+1, 2\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^2*(-ArcSinh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Maple [C] time = 0.061, size = 38, normalized size = 0.6

$$\frac{(\mathrm{Arcsinh}\,(ax))^{2+n}}{a^2\,(2+n)}\,{}_1\mathrm{F}_2\!\left(1+\frac{n}{2};\frac{3}{2},2+\frac{n}{2};(\mathrm{Arcsinh}\,(ax))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^n,x)
```

```
[Out] 1/a^2/(2+n)*arcsinh(a*x)^(2+n)*hypergeom([1+1/2*n],[3/2,2+1/2*n],arcsinh(a*
x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arsinh}\,(ax)^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\,\mathrm{arsinh}\,(ax)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x*arcsinh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{asinh}^n\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**n,x)
```

```
[Out] Integral(x*asinh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^n, x)
```

3.133 $\int \sinh^{-1}(ax)^n\,dx$

Optimal. Leaf size=49

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a}-\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a}$$

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a*(-ArcSinh[a*x])^n) - Gamm
a[1 + n, ArcSinh[a*x]]/(2*a)
```

Rubi [A] time = 0.0467243, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5657, 3307, 2181}

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a}-\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^n,x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a*(-ArcSinh[a*x])^n) - Gamm
a[1 + n, ArcSinh[a*x]]/(2*a)
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n\cosh(x)\,dx, x, \sinh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{2a}+\frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{2a}\\ &= \frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)}{2a}-\frac{\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0251728, size = 45, normalized size = 0.92

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)-\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^n,x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - Gamma[1 +
 n, ArcSinh[a*x]])/(2*a)
```

Maple [C] time = 0.053, size = 40, normalized size = 0.8

$$\frac{(\text{Arcsinh}\,(ax))^{1+n}}{a\,(1+n)}{}_1F_2(\frac{1}{2}+\frac{n}{2};\frac{1}{2},\frac{3}{2}+\frac{n}{2};\frac{(\text{Arcsinh}\,(ax))^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n,x)
```

```
[Out] 1/a/(1+n)*arcsinh(a*x)^(1+n)*hypergeom([1/2+1/2*n],[1/2,3/2+1/2*n],1/4*arcs
inh(a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{arsinh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n,x)
```

```
[Out] Integral(asinh(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n, x)
```

3.134 $\int \frac{\sinh^{-1}(ax)^n}{x} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\sinh^{-1}(ax)^n}{x}, x\right)$$

```
[Out] Unintegrable[ArcSinh[a*x]^n/x, x]
```

Rubi [A] time = 0.0152087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/x, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x} \, dx = \int \frac{\sinh^{-1}(ax)^n}{x} \, dx$$

Mathematica [A] time = 0.276354, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^n}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/x,x]
```

```
[Out] Integrate[ArcSinh[a*x]^n/x, x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{Arcsinh}(ax))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x,x)
```

```
[Out] int(arcsinh(a*x)^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(ax)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^n/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x,x)
```

```
[Out] Integral(asinh(a*x)**n/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/x, x)
```

3.135 $\int \frac{\sinh^{-1}(ax)^n}{x^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\sinh^{-1}(ax)^n}{x^2}, x\right)$$

```
[Out] Unintegrable[ArcSinh[a*x]^n/x^2, x]
```

Rubi [A] time = 0.0164332, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/x^2,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/x^2, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x^2} dx = \int \frac{\sinh^{-1}(ax)^n}{x^2} dx$$

Mathematica [A] time = 0.895444, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^n}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/x^2,x]
```

```
[Out] Integrate[ArcSinh[a*x]^n/x^2, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Arcsinh}\,(ax))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x^2,x)
```

```
[Out] int(arcsinh(a*x)^n/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}\,(ax)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^n/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{asinh}^n\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x**2,x)
```

```
[Out] Integral(asinh(a*x)**n/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/x^2, x)
```

3.136 $\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\text{Erfi}\left(\right.}{48}$$

```
[Out] (x^3*Sqrt[a + b*ArcSinh[c*x]])/3 - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3) + (Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.604637, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\text{Erfi}\left(\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```


```
[Out] (x^3*Sqrt[a + b*ArcSinh[c*x]])/3 - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3) + (Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx = \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{1}{6}(bc)\int \frac{x^3}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$= \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{6c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{(ib)\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{a+bx}} - \frac{i\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{6c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{24c^3} + \frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \right.}{8c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{b\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{48c^3} - \frac{b\,\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{a+bx}}\,dx, x, \right.}{48c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{24c^3} - \frac{\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\right.}{}$$

$$= \frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} + \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}$$

<u>Mathematica</u> [A] time = 0.402476, size = 215, normalized size = 1.01

$$\frac{e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2}, \frac{a}{b}+\sinh^{-1}(cx)\right) + \sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2}, -\frac{3(a+b\sinh}{b}\right.\right.}{72c^3\sqrt{-\frac{(a+b\sin}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Ga
mma[3/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2,
(-3*(a + b*ArcSinh[c*x]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[3/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSin
h[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sq
rt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\mathrm{Arcsinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arsinh}\,(cx) + a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

3.137 $\int x\sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}+\frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] Sqrt[a + b*ArcSinh[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (Sqrt
[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(16*c^2) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(16*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.43199, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}+\frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] Sqrt[a + b*ArcSinh[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (Sqrt
[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(16*c^2) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(16*c^2*E^((2*a)/b))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\sqrt{a+b\sinh^{-1}(cx)}\,dx = \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} - \frac{1}{4}(bc)\int \frac{x^2}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$= \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}$$

$$= \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} + \frac{b\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{4c^2}$$

$$= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8c^2}$$

$$= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16c^2} - \frac{b\,\text{Subs}}{}$$

$$= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} - \frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8c^2} -$$

$$= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)} - \frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2} - \frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A] time = 0.0950965, size = 127, normalized size = 0.88

$$\frac{e^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{3}{2}, -\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right) + e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{3}{2}, \frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{8\sqrt{2}c^2\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-2*(a + b*A
rcSinh[c*x]))/b] + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (
2*(a + b*ArcSinh[c*x]))/b]))/(8*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSi
nh[c*x])^2/b^2)])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x\sqrt{a + b\mathrm{Arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arsinh}\,(cx) + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a + b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

3.138 $\int \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rubi [A] time = 0.250403, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(cx)}\,dx = x\sqrt{a+b\sinh^{-1}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2c}$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} + \frac{b\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c} - \frac{b\,\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}\right.}{4c}$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} + \frac{\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2c} - \frac{\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \right.}{2c}$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] time = 0.224878, size = 101, normalized size = 0.99

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\frac{\mathrm{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}} - \frac{e^{\frac{2a}{b}}\mathrm{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt{a+b\mathrm{Arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

3.139 $\int x^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2} dx$

Optimal. Leaf size=282

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\sqrt{\frac{\pi}{3}}b^{3/2}e^{-\frac{3a}{b}}\text{Er}$$

```
[Out] (b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(6*c) + (x^3*(a + b*ArcSinh[c*x])^(3/2))/3
 - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c
^3) + (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(96*c^3) - (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.862231, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5663, 5758, 5717, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\sqrt{\frac{\pi}{3}}b^{3/2}e^{-\frac{3a}{b}}\text{Er}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```


```
[Out] (b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(6*c) + (x^3*(a + b*ArcSinh[c*x])^(3/2))/3
 - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c
^3) + (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(96*c^3) - (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
```

```
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a+b\sinh^{-1}(cx)\right)^{3/2} dx = \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(bc)\int \frac{x^3\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}} dx$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{1}{12}b^2\int \frac{x^2}{\sqrt{a+b\sinh^{-1}}$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(\right.$$

Mathematica [A] time = 0.287188, size = 215, normalized size = 0.76

$$-\frac{be^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\mathrm{Gamma}\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\mathrm{Gamma}\left(\frac{5}{2},-\frac{3(a}{}\right.\right.}{216c^3\sqrt{-\frac{(}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -(b*Sqrt[a + b*ArcSinh[c*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b
)]*Gamma[5/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
```

```
5/2, (-3*(a + b*ArcSinh[c*x]))/b] - 27*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]
*Gamma[5/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*
ArcSinh[c*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c*x]))/b]))/(216*c^3*E^((3*a
)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int x^{2} \left(a+b \mathrm{Arcsinh}\left(c x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}\left(cx\right) + a)^{\frac{3}{2}} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(a+b\operatorname{asinh}(cx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{arsinh}(cx)+a\right)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x^2, x)
```

3.140 $\int x\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=179

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{3bx\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\,\mathrm{si}\right.}{}$$

```
[Out] (-3*b*x*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(8*c) + (a + b*ArcSinh[
c*x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(3/2))/2 - (3*b^(3/2)*E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*c^2)
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.478203, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5663, 5758, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{3bx\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\,\mathrm{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*x*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(8*c) + (a + b*ArcSinh[
c*x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(3/2))/2 - (3*b^(3/2)*E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*c^2)
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*c^2*E^((2*a)/b))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(cx)\right)^{3/2}dx=\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\frac{1}{4}(3bc)\int\frac{x^2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}dx$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{16}\left(3b^2\right)\int\frac{x}{\sqrt{a+b\sinh\cdots}}$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\cdots$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\cdots$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\cdots$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\cdots$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\cdots$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\cdots$$

Mathematica [A] time = 0.100346, size = 129, normalized size = 0.72

$$\frac{be^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{5}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{5}{2},-\frac{2\left(a+b\sinh^{-1}(cx\right.}{b}\right.\right.}{16\sqrt{2}c^2\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (b*Sqrt[a + b*ArcSinh[c*x]]*(-(Sqrt[a/b + ArcSinh[c*x]]*Gamma[5/2, (-2*(a +
 b*ArcSinh[c*x]))/b]) + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[5
/2, (2*(a + b*ArcSinh[c*x]))/b]))/(16*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[-((a + b
*ArcSinh[c*x])^2/b^2)])
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x\left(a+b\text{Arcsinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{asinh}(cx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\operatorname{arsinh}(cx)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

3.141 $\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(\right.$$

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rubi [A] time = 0.254633, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```


```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```


Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\sinh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{1}{4}\left(3b^2\right)\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx}}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,}{4c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,}{8c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{}\right.}{4c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c}}{\sqrt{b}}\right.}{8c}$$

Mathematica [A] time = 1.03352, size = 251, normalized size = 1.86

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\frac{\mathrm{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}} - \frac{e^{\frac{2a}{b}}\mathrm{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}\right)}{2c} + \frac{\sqrt{b}\left(4\sqrt{b}\left(2cx\sinh^{-1}(cx)-3\sqrt{c^2x^2+1}\right)\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{asinh}\,(cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

3.142 $\int x^2 \left(a+b\sinh^{-1}(cx)\right)^{5/2} \, dx$

Optimal. Leaf size=327

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}+\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3}+\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{-}}{}$$

```
[Out] (-5*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/(6*c^2) + (5*b^2*x^3*Sqrt[a + b*ArcSinh
[c*x]])/36 + (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(9*c^3) - (
5*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(18*c) + (x^3*(a + b*
ArcSinh[c*x])^(5/2))/3 - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSin
h[c*x]]/Sqrt[b]])/(64*c^3) + (5*b^(5/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3) + (15*b^(5/2)*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*c^3*E^(a/b)) - (5*b^(5/2)*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.25032, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}+\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3}+\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-5*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/(6*c^2) + (5*b^2*x^3*Sqrt[a + b*ArcSinh
[c*x]])/36 + (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(9*c^3) - (
5*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(18*c) + (x^3*(a + b*
ArcSinh[c*x])^(5/2))/3 - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSin
h[c*x]]/Sqrt[b]])/(64*c^3) + (5*b^(5/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3) + (15*b^(5/2)*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*c^3*E^(a/b)) - (5*b^(5/2)*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3*E^((3*a)/b))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^2 \left(a+b\sinh^{-1}(cx)\right)^{5/2} dx = \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{5/2} - \frac{1}{6}(5bc)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\sqrt{1+c^2x^2}}dx$$

$$= -\frac{5bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{18c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{5/2} + \frac{1}{12}\left(5b^2\right)\int x^2\sqrt{a$$

$$= \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x^2}\left(a+b\right.}{18c}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c^3}$$

Mathematica [A] time = 0.431815, size = 215, normalized size = 0.66

$$-\frac{e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(cx)\right)^{5/2}\left(81e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{7}{2},-\frac{3(a\right.\right.}{648c^3\left(-\frac{(a+l}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^(5/2)*(81*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)
]*Gamma[7/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7
/2, (-3*(a + b*ArcSinh[c*x]))/b] - 81*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[7/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*A
rcSinh[c*x])/b)]*Gamma[7/2, (3*(a + b*ArcSinh[c*x]))/b]))/(648*c^3*E^((3*a)
/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(3/2))
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x^2, x)
```

3.143 $\int x\left(a+b\sinh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=223

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}+\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] (15*b^2*Sqrt[a + b*ArcSinh[c*x]])/(64*c^2) + (15*b^2*x^2*Sqrt[a + b*ArcSinh
[c*x]])/32 - (5*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(8*c) + (
a + b*ArcSinh[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(5/2))/2 - (1
5*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(256*c^2) - (15*b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(256*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.750858, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}+\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c*x])^(5/2),x]
```


```
[Out] (15*b^2*Sqrt[a + b*ArcSinh[c*x]])/(64*c^2) + (15*b^2*x^2*Sqrt[a + b*ArcSinh
[c*x]])/32 - (5*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(8*c) + (
a + b*ArcSinh[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(5/2))/2 - (1
5*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(256*c^2) - (15*b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(256*c^2*E^((2*a)/b))
```


Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(cx)\right)^{5/2}dx=\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{5/2}-\frac{1}{4}(5bc)\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\sqrt{1+c^2x^2}}dx$$

$$=-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{5/2}+\frac{1}{16}\left(15b^2\right)\int x\sqrt{a+b}$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}{4c^2}+$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}{4c^2}+$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}{4c^2}+$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}$$

Mathematica [A] time = 0.0720903, size = 115, normalized size = 0.52

$$\frac{e^{-\frac{2a}{b}}\left(b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\mathrm{Gamma}\left(\frac{7}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-b^3\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\mathrm{Gamma}\left(\frac{7}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{32\sqrt{2}c^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-(b^3*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh[c*x]))
/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7/2, (2*(a + b*ArcSin
h[c*x]))/b])/(32*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x, x)
```

3.144 $\int \left(a+b\sinh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=155

$$\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}+\frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{c^2x^2+1}\left(a+b\,\mathrm{s}\right.}{2c}$$

```
[Out] (15*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/4 - (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^(3/2))/(2*c) + x*(a + b*ArcSinh[c*x])^(5/2) + (15*b^(5/2)*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c) - (15*b^(5/2)*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c*E^(a/b))
```

Rubi [A] time = 0.406392, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}+\frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{c^2x^2+1}\left(a+b\,\mathrm{s}\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(5/2),x]
```



```
[Out] (15*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/4 - (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^(3/2))/(2*c) + x*(a + b*ArcSinh[c*x])^(5/2) + (15*b^(5/2)*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c) - (15*b^(5/2)*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c*E^(a/b))
```


Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{5/2} dx = x\left(a+b\sinh^{-1}(cx)\right)^{5/2} - \frac{1}{2}(5bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\sqrt{1+c^2x^2}} dx$$

$$= -\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{5/2} + \frac{1}{4}\left(15b^2\right)\int \sqrt{a+b\sinh^{-}}$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)} - \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{5/2} -$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)} - \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{5/2} -$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)} - \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{5/2} +$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)} - \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{5/2} +$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)} - \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{5/2} +$$

Mathematica [A] time = 3.10819, size = 282, normalized size = 1.82

$$\sqrt{b}e^{-\frac{a}{b}}\left(\frac{4\sqrt{b}\left(-2a^2e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)-2a^2\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)+e^{a/b}\left(a+b\sinh^{-1}(cx)\right)\left(5\left(3bcx-2a\sqrt{c^2x}\right.\right.\right.}{\sqrt{a+b\sinh^{-1}(cx)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (Sqrt[b]*(-((4*a^2 - 15*b^2)*E^((2*a)/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]) + (4*a^2 - 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqr
t[b]] + (4*Sqrt[b]*(E^(a/b)*(a + b*ArcSinh[c*x])*(5*(3*b*c*x - 2*a*Sqrt[1 +
 c^2*x^2]) + 2*(4*a*c*x - 5*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 4*b*c*x*Arc
Sinh[c*x]^2) - 2*a^2*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, a/b +
ArcSinh[c*x]] - 2*a^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, -((a + b*A
rcSinh[c*x])/b)]))/Sqrt[a + b*ArcSinh[c*x]]))/(16*c*E^(a/b))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(cx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(cx) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(cx) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2), x)
```

3.145 $\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3) +
 (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
8*Sqrt[b]*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[
b]*c^3*E^(a/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.350143, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5669, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3) +
 (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
8*Sqrt[b]*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[
b]*c^3*E^(a/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}} + \frac{\cosh(3x)}{4\sqrt{a+bx}}\right) dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^3}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx\right.}{8c^3}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{4bc^3} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{4bc^3} - \text{Su}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \cdots$$

Mathematica [A] time = 0.216801, size = 196, normalized size = 1.01

$$\frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-3e^{\frac{2a}{b}}\sqrt{-\frac{a+}{}}\cdots\right.}{24c^3\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sq
rt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/
b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*
(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a+b\text{Arcsinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b \operatorname{arsinh}(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + b \operatorname{asinh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

3.146 $\int \frac{x}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

```
[Out] -(E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
4*Sqrt[b]*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.179085, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -(E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
4*Sqrt[b]*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^2*E^((2*a)/b))
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2bc^2} + \frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2bc^2}$$
$$= -\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

Mathematica [A] time = 0.0646644, size = 108, normalized size = 1.01

$$\frac{e^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2}, -\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right) + e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2}, \frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{4\sqrt{2}c^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]
)/(4*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+b\text{Arcsinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b \operatorname{arsinh}(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + b \operatorname{asinh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*asinh(c*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b\,\mathrm{arsinh}\,(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

3.147 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rubi [A] time = 0.104205, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}\\
&= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc}\\
&= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}\\
&= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}
\end{aligned}$$

Mathematica [A] time = 0.0964812, size = 101, normalized size = 1.15

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2c\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\mathrm{Arcsinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

3.148 $\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (E^((3*a)/b)*Sq
rt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) -
 (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b))
+ (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*
c^3*E^((3*a)/b))
```

Rubi [A] time = 0.323865, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5665, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```


```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (E^((3*a)/b)*Sq
rt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) -
 (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b))
+ (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*
c^3*E^((3*a)/b))
```


Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
```

```
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{2\,\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{3\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx, x, \sin\right.}{2bc^3}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4bc^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}\right.}{4bc^3}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2b^2c^3} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{2b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{-\frac{a}{b}}}{}$$

Mathematica [A] time = 0.37857, size = 290, normalized size = 1.28

$$\frac{e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}e^{3\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c*x]) + E^((3*a)/b + 4*ArcSinh[c*x])
 - E^((3*a)/b + 6*ArcSinh[c*x]) - E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + A
rcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - E^(
(2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a +
 b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \mathrm{Arcsinh}\left(c x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(b \operatorname{arsinh}\left(c x\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b\operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.149 $\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^((2*a)/b)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2) + (Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2*E^(
(2*a)/b))
```

Rubi [A] time = 0.161703, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c*x])^(3/2),x]
```


```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^((2*a)/b)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2) + (Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2*E^(
(2*a)/b))
```


Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx &= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c^2}+\frac{2\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\cdots}\right.}{}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}
\end{aligned}$$

Mathematica [A] time = 0.108916, size = 134, normalized size = 0.99

$$\frac{e^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-\sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-2e^{\frac{2a}{b}}\sin$$

$$2bc^2\sqrt{a+b\sinh^{-1}(cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]
))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*A
rcSinh[c*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c*x]])/(2*b*c^2*E^((2*a)/b)
*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int x\,(a+b\text{Arcsinh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.150 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rubi [A] time = 0.255644, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}+\frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c}+\frac{2\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x}{b}}\right.}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

Mathematica [A] time = 0.225622, size = 137, normalized size = 1.18

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(e^{\frac{2a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+e^{\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\mathrm{s}}{}\right.\right.}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

3.151 $\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}$$

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (8*x)/(3*b^
2*c^2*Sqrt[a + b*ArcSinh[c*x]]) - (4*x^3)/(b^2*Sqrt[a + b*ArcSinh[c*x]]) -
(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2)*c^3) +
(E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2
*b^(5/2)*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2
)*c^3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(2*b^(5/2)*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.899481, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205, 5657}

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (8*x)/(3*b^
2*c^2*Sqrt[a + b*ArcSinh[c*x]]) - (4*x^3)/(b^2*Sqrt[a + b*ArcSinh[c*x]]) -
(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2)*c^3) +
(E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2
*b^(5/2)*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2
)*c^3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(2*b^(5/2)*c^3*E^((3*a)/b))
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{4\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(2c)\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}}{b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{12\int \frac{}{\sqrt{a+}}}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{8\,\mathrm{Subst}\,(}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{4\,\mathrm{Subst}\,(}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{8\,\mathrm{Subst}\,(}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{4e^{a/b}\sqrt{\pi}\mathrm{e}}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{4e^{a/b}\sqrt{\pi}\mathrm{e}}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\mathrm{er}}{}$$

Mathematica [A] time = 1.5089, size = 340, normalized size = 1.25

$$\frac{e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-6\sqrt{3}be^{3\sinh^{-1}(cx)}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+2be^{\frac{2a}{b}+3\sinh^{-1}(cx)}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (2*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x
])*Gamma[1/2, a/b + ArcSinh[c*x]] - 6*Sqrt[3]*b*E^(3*ArcSinh[c*x])*(-((a +
b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 2*b*E^(
(2*a)/b + 3*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a
 + b*ArcSinh[c*x])/b)] - E^((3*a)/b)*((-1 + E^(2*ArcSinh[c*x]))*(b*(-1 + E^
(4*ArcSinh[c*x])) + a*(6 + 4*E^(2*ArcSinh[c*x]) + 6*E^(4*ArcSinh[c*x])) + 2
*b*(3 + 2*E^(2*ArcSinh[c*x]) + 3*E^(4*ArcSinh[c*x]))*ArcSinh[c*x]) + 6*Sqrt
[3]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x]
)*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(12*b^2*c^3*E^(3*(a/b + ArcSinh[
c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b\mathrm{Arcsinh}\left(cx\right)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(b\,\mathrm{arsinh}\left(cx\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+b\operatorname{asinh}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

3.152 $\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}-\frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - 4/(3*b^2*c^2*
Sqrt[a + b*ArcSinh[c*x]]) - (8*x^2)/(3*b^2*Sqrt[a + b*ArcSinh[c*x]]) - (2*E
^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(3*b
^(5/2)*c^2) + (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(3*b^(5/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.517266, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5667, 5774, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5675}

$$-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}-\frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - 4/(3*b^2*c^2*
Sqrt[a + b*ArcSinh[c*x]]) - (8*x^2)/(3*b^2*Sqrt[a + b*ArcSinh[c*x]]) - (2*E
^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(3*b
^(5/2)*c^2) + (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(3*b^(5/2)*c^2*E^((2*a)/b))
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
```

```
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{2\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(4c)\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/}}}{3b}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{16\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{16\,\mathrm{Subs}}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{16\,\mathrm{Subs}}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{8\,\mathrm{Subst}}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4\,\mathrm{Subst}}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8\,\mathrm{Subst}}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2e^{\frac{2a}{b}}\sqrt{2}}{}$$

Mathematica [A] time = 0.709698, size = 200, normalized size = 1.09

$$\frac{e^{-2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(e^{\frac{2a}{b}}\left(4\sqrt{2}e^{2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\left(a+b\sinh^{-1}(cx)\right)\mathrm{Gamma}\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-4ae^{4\sinh^{-1}(cx}\right.\right.}{6b^2c^2\left(a+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*b*E^(2*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/
2, (-2*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*a*E^(4*ArcSinh[
c*x]) - b*E^(4*ArcSinh[c*x]) - 4*b*(1 + E^(4*ArcSinh[c*x]))*ArcSinh[c*x] +
4*Sqrt[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSin
h[c*x])*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]))/(6*b^2*c^2*E^(2*(a/b + Arc
Sinh[c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\operatorname{arsinh}\left(cx\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

3.153 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\sqrt{c^2x^2+1}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (4*x)/(3*b^2*Sq
rt[a + b*ArcSinh[c*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(3*b^(5/2)*c) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
])/(3*b^(5/2)*c*E^(a/b))
```

Rubi [A] time = 0.272854, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5655, 5774, 5657, 3307, 2180, 2205, 2204}

$$\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\sqrt{c^2x^2+1}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-5/2),x]
```


```
[Out] (-2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (4*x)/(3*b^2*Sq
rt[a + b*ArcSinh[c*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(3*b^(5/2)*c) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
])/(3*b^(5/2)*c*E^(a/b))
```


Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{4\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{3b^2}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{4\,\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sin$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{4\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}} + \frac{2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2e^{-\frac{a}{b}}\sqrt{}$$

Mathematica [A] time = 0.577197, size = 181, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(-2be^{\sinh^{-1}(cx)}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right) - 2e^{\frac{2a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\left(a+b\sinh^{-1}\right.\right.}{3b^2c\left(a+b\sin\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c*x])) - 2*b*ArcSinh[c*x] + b*E^(2*A
rcSinh[c*x])*(1 + 2*ArcSinh[c*x]))) - 2*E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*Gamma[1/2, a/b + ArcSinh[c*x]] - 2*b*
E^ArcSinh[c*x]*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSin
h[c*x])/b)])/(3*b^2*c*E^((a + b*ArcSinh[c*x])/b)*(a + b*ArcSinh[c*x])^(3/2)
```

)

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}(cx))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

3.154 $\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=346

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}-\frac{3\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}$$

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (8*x)/(15*b
^2*c^2*(a + b*ArcSinh[c*x])^(3/2)) - (4*x^3)/(5*b^2*(a + b*ArcSinh[c*x])^(3
/2)) - (16*Sqrt[1 + c^2*x^2])/(15*b^3*c^3*Sqrt[a + b*ArcSinh[c*x]]) - (24*x
^2*Sqrt[1 + c^2*x^2])/(5*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3) - (3*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3)
 - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3*E^(a/b
)) + (3*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(
7/2)*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.03265, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5667, 5774, 5665, 3308, 2180, 2204, 2205, 5655, 5779}

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}-\frac{3\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (8*x)/(15*b
^2*c^2*(a + b*ArcSinh[c*x])^(3/2)) - (4*x^3)/(5*b^2*(a + b*ArcSinh[c*x])^(3
/2)) - (16*Sqrt[1 + c^2*x^2])/(15*b^3*c^3*Sqrt[a + b*ArcSinh[c*x]]) - (24*x
^2*Sqrt[1 + c^2*x^2])/(5*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3) - (3*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3)
 - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3*E^(a/b
)) + (3*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(
```

```
7/2)*c^3*E^((3*a)/b))
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} + \frac{4\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx}{5bc} + \frac{(6c)\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}}{5b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{12}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{}{15b}$$

Mathematica [A] time = 1.56026, size = 417, normalized size = 1.21

$$\frac{e^{-\sinh^{-1}(cx)}\left(-4e^{\frac{a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\left(a+b\sinh^{-1}(cx)\right)^2\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+4a^2+2b(4a-b)\sinh^{-1}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (3*b^2*E^ArcSinh[c*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*ArcSinh[c*x]
 + 4*b^2*ArcSinh[c*x]^2 - 4*E^(a/b + ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]
*(a + b*ArcSinh[c*x])^2*Gamma[1/2, a/b + ArcSinh[c*x]])/E^ArcSinh[c*x] - 3*
```

```
(b^2*E^(3*ArcSinh[c*x]) + (2*(a + b*ArcSinh[c*x])*(E^(3*(a/b + ArcSinh[c*x]
))*(6*a + b + 6*b*ArcSinh[c*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c*x])/b))^(
3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b]))/E^((3*a)/b)) + (2*(a + b*Arc
Sinh[c*x])*(E^(a/b + ArcSinh[c*x])*(2*a + b + 2*b*ArcSinh[c*x]) + 2*b*(-((a
 + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]))/E^(a/b
) - (3*(b^2 + 2*(a + b*ArcSinh[c*x])*(6*a - b + 6*b*ArcSinh[c*x] - 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*
Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])))/E^(3*ArcSinh[c*x]))/(60*b^3*c^3*(
a + b*ArcSinh[c*x])^(5/2))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \operatorname{Arcsinh}\left(c x\right)\right)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(b \operatorname{arsinh}\left(c x\right) + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b \operatorname{arsinh}(cx) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(7/2), x)
```

3.155 $\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=219

$$\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}-\frac{32x\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4}{15b^2c^2\left(a+b\sinh^{-1}(c\right.}$$

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - 4/(15*b^2*c^2
*(a + b*ArcSinh[c*x])^(3/2)) - (8*x^2)/(15*b^2*(a + b*ArcSinh[c*x])^(3/2))
- (32*x*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (8*E^((2*a
)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(15*b^(7/2
)*c^2) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(1
5*b^(7/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.505574, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5667, 5774, 5665, 3307, 2180, 2204, 2205, 5675}

$$\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}-\frac{32x\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4}{15b^2c^2\left(a+b\sinh^{-1}(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - 4/(15*b^2*c^2
*(a + b*ArcSinh[c*x])^(3/2)) - (8*x^2)/(15*b^2*(a + b*ArcSinh[c*x])^(3/2))
- (32*x*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (8*E^((2*a
)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(15*b^(7/2
)*c^2) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(1
5*b^(7/2)*c^2*E^((2*a)/b))
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} + \frac{2\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx}{5bc} + \frac{(4c)\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/}}}{5b}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} +$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} -$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} -$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} -$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} -$$

Mathematica [A] time = 1.13849, size = 208, normalized size = 0.95

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)\left(e^{-\frac{2a}{b}}\left(8\sqrt{2}b\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+2e^{2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(4a+4b\sinh^{-1}(cx\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])*((2*E^(2*(a/b + ArcSinh[c*x]))*(4*a + b + 4*b*ArcSin
h[c*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-2*(a +
 b*ArcSinh[c*x]))/b])/E^((2*a)/b) + (-8*a + 2*b - 8*b*ArcSinh[c*x] + 8*Sqrt
[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x]
)*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b])/E^(2*ArcSinh[c*x])) + 3*b^2*Sinh[
2*ArcSinh[c*x]])/(15*b^3*c^2*(a + b*ArcSinh[c*x])^(5/2))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x \left(a + b \mathrm{Arcsinh}\left(cx\right)\right)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b \operatorname{arsinh}(cx) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(7/2), x)
```

3.156 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{8\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-$$

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (4*x)/(15*b^2*(
a + b*ArcSinh[c*x])^(3/2)) - (8*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*Arc
Sinh[c*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(1
5*b^(7/2)*c) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7
/2)*c*E^(a/b))
```

Rubi [A] time = 0.447322, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-7/2),x]
```


```
[Out] (-2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (4*x)/(15*b^2*(
a + b*ArcSinh[c*x])^(3/2)) - (8*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*Arc
Sinh[c*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(1
5*b^(7/2)*c) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7
/2)*c*E^(a/b))
```


Rule *5655*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} + \frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx}{5b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{4\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(8c)\int}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}} + \frac{8\,\mathrm{Sub}}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4\,\mathrm{Sub}}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8\,\mathrm{Sub}}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4e^{a/b}}{}$$

Mathematica [A] time = 0.583096, size = 210, normalized size = 1.18

$$\frac{8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\left(a+b\sinh^{-1}(cx)\right)^2\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right) - 4e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(cx)\right)\left(2b\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c*x]) + b^2*(3 -
 2*ArcSinh[c*x] + 4*ArcSinh[c*x]^2)))/E^ArcSinh[c*x] + 8*E^(a/b)*Sqrt[a/b +
 ArcSinh[c*x]]*(a + b*ArcSinh[c*x])^2*Gamma[1/2, a/b + ArcSinh[c*x]] - (4*(
```

```
a + b*ArcSinh[c*x])*(E^(a/b + ArcSinh[c*x])*(2*a + b + 2*b*ArcSinh[c*x]) +
2*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]
))/E^(a/b))/(30*b^3*c*(a + b*ArcSinh[c*x])^(5/2))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(7/2),x)

[Out] int(1/(a+b*arcsinh(c*x))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{arsinh}\,(cx) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")

[Out] integrate((b*arcsinh(c*x) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-7/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
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        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
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        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

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#Dec 24, 2019. Nasser: Ported original Maple grading function by
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#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
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    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```